Admissibility Fibrations: Policy-Indexed Identity, Certified Quotients, and Lyapunov-Stable Emergence

Douglas C. Youvan

doug@youvan.com

www.youvan.ai

July 6, 2026

We formulate Admissibility Algebra as an indexed ∞-categorical theory of policy-relative identity. The primitive datum is not an object, morphism, path, or equivalence, but a policy determining which transformations preserve which invariants under which certificates and coherence laws. Formally, an admissibility algebra is modeled as a Grothendieck fibration $p : \mathfrak{E} \to \mathfrak{P}$ over a bicategory or locally ordered ∞-category of policies. The fiber $\mathfrak{E}\pi$ over a policy π contains presentations equipped with π-orbits, π-stabilizers, π-invariants, certificate spaces, and higher coherence. The primitive judgment is X ≡π Y, defined by inhabitation of Certπ(X,Y). Ordinary equality, isomorphism, homotopy equivalence, gauge equivalence, observational equivalence, and biological continuity arise as special fibers or policies. Quotients Ω/π are fiberwise colimits or homotopy quotients. A dynamical enrichment assigns to each policy π a Lyapunov-type functional Vπ whose zero set is a π-orbit rather than a point. Emergence is then defined as nontrivial, Lyapunov-stable admissibility quotient formation. A finite model is supplied by the standard genetic code, treated as a policy-object $\mathcal{G}$std = ($\mathcal{C}$, ϕ, D, Q, o) with translation quotient, rank-10 projection, and ordered-cube topology.

Keywords: Admissibility Algebra, admissibility fibration, policy-relative identity, indexed ∞-category, Grothendieck fibration, policy bicategory, certified transformation, invariant preservation, homotopy quotient, Lyapunov stability, stable quotient formation, emergence, genetic code algebra, rank-10 projection, G-A-C-U topology.

A collaboration with OpenAI GPT-5.5 Thinking. CC4.0.

https://orcid.org/0009-0004-4206-4253

1. Introduction

1.1 Problem

Let Ω be a universe of presentations. A presentation may be a term, diagram, state, field, codeword, graph, type, model, process-stage, or context-indexed description. Let T be a universe of candidate transformations between presentations. A transformation $t : X \to Y$ need not preserve identity. It is only a candidate transition.

Classical equality suppresses the jurisdiction under which sameness is asserted. Category theory begins after a class of morphisms has already been chosen. Homotopy type theory begins after identity types or path spaces have already been determined. Gauge theory begins after gauge transformations have been distinguished from physically effective transformations. Quotient constructions begin after an equivalence relation, group action, congruence, localization, or descent datum has already been specified.

The missing datum is the rule by which transformations acquire identity force.

We call such a rule an admissibility policy.

1.2 Primitive judgment

For presentations $X,Y \in \Omega$ and a policy π, write

$X \equiv_\pi Y$

to mean that X and Y are identical under π.

The subscript π is not decoration. It is part of the identity judgment.

The intended reading is:

$X \equiv_\pi Y$ iff there exists a π-admissible, invariant-preserving, coherently certified transformation from X to Y.

Thus identity is indexed by admissibility.

1.3 Aim

The aim is to define a mathematical structure in which policies are first-class objects, identity judgments are policy-indexed, certificates are proof-relevant, quotient formation is fiberwise, and stability is measured relative to policy-orbits.

The proposed formal home is an indexed ∞-category

$\mathfrak{U} : \mathfrak{P}^{op} \to \mathrm{Cat}_\infty$

or equivalently a Grothendieck fibration

$p : \mathfrak{E} \to \mathfrak{P}$,

where $\mathfrak{P}$ is a bicategory, double category, quantaloid, or locally ordered ∞-category of policies.

The fiber over π contains the identity regime licensed by π.

1.4 Thesis

Admissibility Algebra is not a replacement for category theory or homotopy type theory. It is a regulatory layer over them.

A category is obtained after a policy has selected admissible arrows.

A homotopy type is obtained after a policy has selected admissible paths.

A quotient is obtained after a policy has selected admissible collapses.

A stable emergent object is obtained after a policy quotient has acquired Lyapunov stability.

In compressed form:

objecthood = presentation under policy

identity = certified invariant-preserving transformation

quotient = policy-indexed collapse

stability = Lyapunov control of policy-tension

emergence = stable admissibility quotient formation

2. Admissibility Systems

2.1 Basic data

An admissibility system is a tuple

$A = (\Omega, T, \Pi, Adm, Inv, Cert, Coh)$

where:

Ω is a universe of presentations.

T is a universe of candidate transformations.

Π is a universe of admissibility policies.

Adm assigns to each $\pi \in \Pi$ and $X,Y \in \Omega$ a class, type, or space

$\mathrm{Adm}_\pi(X,Y) \subseteq T(X,Y)$.

Inv assigns to each $\pi \in \Pi$ and $X \in \Omega$ an invariant object

$\mathrm{Inv}_\pi(X)$.

Cert assigns to each $\pi \in \Pi$ and $X,Y \in \Omega$ a certificate object

$\mathrm{Cert}_\pi(X,Y)$.

Coh assigns higher coherence laws among certificates.

The primitive identity judgment is

$X \equiv_\pi Y \Leftrightarrow \mathrm{Cert}_\pi(X,Y)$ is inhabited.

2.2 Certificates

A certificate $c \in \mathrm{Cert}_\pi(X,Y)$ consists minimally of data

$c = (t, a, i, h)$

where:

$t \in T(X,Y)$,

$a : t \in \mathrm{Adm}_\pi(X,Y)$,

$i : t_{(*)}\mathrm{Inv}_\pi(X) \cong \mathrm{Inv}_\pi(Y)$,

$h : \mathrm{Coh}_\pi(t,i)$.

Thus a certificate witnesses not merely the existence of a transformation, but its admissibility, invariant preservation, and coherence.

If $\mathrm{Cert}_\pi(X,Y)$ is proposition-valued, the identity theory is proof-irrelevant.

If $\mathrm{Cert}_\pi(X,Y)$ is set-valued, multiple witnesses may exist.

If $\mathrm{Cert}_\pi(X,Y)$ is groupoid-valued, certificates have isomorphisms between witnesses.

If $\mathrm{Cert}_\pi(X,Y)$ is ∞-groupoid-valued, identity is homotopical.

If $\mathrm{Cert}_\pi(X,Y)$ is directed, identity becomes process-sensitive and may fail symmetry.

2.3 Policy-objects

A policy-object is not a bare presentation. It is a presentation equipped with identity jurisdiction.

For $X \in \Omega$ and $\pi \in \Pi$, define

$X\pi = (X, \pi, \mathrm{Orb}\pi(X), \mathrm{Stab}\pi(X), \mathrm{Inv}\pi(X), \mathrm{Cert}\pi, \mathrm{Coh}\pi)$.

Here

$\mathrm{Orb}\pi(X) = \{Y \in \Omega : X \equiv\pi Y\}$

and

$\mathrm{Stab}\pi(X) = \{t \in \mathrm{Adm}\pi(X,X) : t \text{ preserves } \mathrm{Inv}\pi(X) \text{ and satisfies } \mathrm{Coh}\pi\}$.

$\mathrm{Orb}\pi(X)$ is the π-identity cloud of X.

$\mathrm{Stab}\pi(X)$ is the π-self-preserving transformation object.

If certificates are proof-relevant, $\mathrm{Orb}\pi(X)$ is not merely a set. It is a groupoid, ∞-groupoid, category, or ∞-category according to the certificate geometry.

2.4 Nonidentity of policy-objects

Proposition 2.4.1. The same presentation may determine distinct policy-objects under distinct policies.

Proof. Let $X \in \Omega$ and let $\pi,\rho \in \Pi$. If $\mathrm{Orb}\pi(X) \neq \mathrm{Orb}\rho(X)$, or $\mathrm{Stab}\pi(X) \neq \mathrm{Stab}\rho(X)$, or $\mathrm{Inv}\pi(X) \neq \mathrm{Inv}\rho(X)$, or $\mathrm{Cert}\pi \neq \mathrm{Cert}\rho$, then $X\pi \neq X\rho$. Such policies exist whenever two identity regimes preserve different invariants or admit different transformations. Therefore the bare presentation X does not determine a unique object independently of policy.

Corollary 2.4.2. Objecthood is policy-relative.

2.5 Classical equality as a policy

Let $\pi\mathrm{disc}$ be the discrete policy:

$\mathrm{Adm}\pi\mathrm{disc}(X,Y) = \{\mathrm{id}X\}$ if $X = Y$,

$\mathrm{Adm}\pi\mathrm{disc}(X,Y) = \emptyset$ otherwise.

Then

$X \equiv\pi\mathrm{disc}\ Y \Leftrightarrow X = Y$.

Thus ordinary equality is recovered as a special policy.

Let $\pi\mathrm{iso}$ be an isomorphism policy in a category C. Then

$X \equiv\pi\mathrm{iso}\ Y \Leftrightarrow \mathrm{Iso}C(X,Y)$ is inhabited.

Let $\pi\mathrm{heq}$ be a homotopy-equivalence policy on spaces. Then

$X \equiv_{\pi\text{heq}} Y \Leftrightarrow X \simeq Y$.

Let π_{gauge} be a gauge policy. Then

$X \equiv_{\pi\text{gauge}} Y \Leftrightarrow$ X and Y lie in the same gauge orbit.

Thus equality, isomorphism, homotopy equivalence, gauge equivalence, and observational equivalence are not abolished. They are indexed.

3. Policy Spaces

3.1 Refinement order

Define a preorder on policies by

$\pi \leq \rho \Leftrightarrow \forall X,Y,\ X \equiv_\pi Y \Rightarrow X \equiv_\rho Y$.

Read $\pi \leq \rho$ as π is stricter than ρ.

Equivalently, in proof-relevant form,

$\pi \leq \rho \Leftrightarrow$ there exists a natural map $\text{Cert}_\pi(X,Y) \to \text{Cert}_\rho(X,Y)$.

Proposition 3.1.1. If $\pi \leq \rho$, then $\text{Orb}_\pi(X) \subseteq \text{Orb}_\rho(X)$.

Proof. If $Y \in \text{Orb}_\pi(X)$, then $X \equiv_\pi Y$. Since $\pi \leq \rho$, $X \equiv_\rho Y$. Hence $Y \in \text{Orb}_\rho(X)$.

Thus stricter policies produce smaller identity clouds.

3.2 Why a preorder is insufficient

The refinement preorder records only implication among identity judgments. It loses certificate multiplicity, directedness, context dependence, partial composition, higher coherence, and obstruction data.

Therefore Π should be regarded as the 1-truncation of a richer policy object $\mathfrak{P}$.

The minimal useful structure is a bicategory of policies.

3.3 Policy bicategory

Let $\mathfrak{P}$ be a bicategory.

Objects of $\mathfrak{P}$ are contexts, regimes, scales, theories, languages, frames, or identity environments.

A 1-morphism

$\pi : a \to b$

is an admissibility policy translating presentations from context a to context b.

A 2-morphism

$\alpha : \pi \Rightarrow \rho$

is a refinement, weakening, comparison, or certificate-transforming map between policies.

Composition

$\rho \circ \pi : a \to c$

is policy composition.

The identity 1-morphism

$id_a : a \to a$

is the strict self-policy at context a.

If each hom-category $\mathfrak{P}(a,b)$ is a preorder, then $\mathfrak{P}$ is locally ordered.

If each hom-category has joins and composition preserves them, $\mathfrak{P}$ is quantaloid-like.

If 2-morphisms carry higher witnesses, $\mathfrak{P}$ is an ∞-bicategory.

3.4 Policy composition

Given policies

$\pi : a \to b$

and

$\rho : b \to c$,

their composite

$\rho \circ \pi : a \to c$

licenses a transformation $X \to Z$ when there exists Y such that X transforms admissibly to Y under π and Y transforms admissibly to Z under ρ, with coherent transport of invariants.

Formally,

$Cert_{\rho\circ\pi}(X,Z)$

contains triples

$(Y, c_\pi \in Cert_\pi(X,Y), c_\rho \in Cert_\rho(Y,Z))$

modulo coherence.

Composition may be partial. It may fail if invariants are incompatible, if certificates cannot be composed, or if the intermediate context is not admissible.

3.5 Meet

When defined, the meet $\pi \wedge \rho$ is the simultaneous policy satisfying

$X \equiv_{\pi \wedge \rho} Y \Leftrightarrow X \equiv_{\pi} Y$ and $X \equiv_{\rho} Y$.

Certificate-theoretically,

$\mathrm{Cert}_{\pi \wedge \rho}(X,Y) \simeq \mathrm{Cert}_{\pi}(X,Y) \times \mathrm{Cert}_{\rho}(X,Y)$

subject to joint coherence.

Invariant-theoretically,

$\mathrm{Inv}_{\pi \wedge \rho}(X) = \mathrm{Inv}_{\pi}(X) \times \mathrm{Inv}_{\rho}(X)$

up to compatibility.

Thus $\pi \wedge \rho$ is stricter than π and ρ.

3.6 Join

When defined, the join $\pi \vee \rho$ is the weakest coherent policy σ satisfying

$\pi \leq \sigma$

and

$\rho \leq \sigma$.

It is generated by transformations admissible under π or ρ, closed under the required policy-composition rules.

Join is not assumed total.

A join may fail because the two policies protect incompatible invariants.

A join may exist but collapse.

3.7 Collapse

A policy π collapses Ω if

$\forall X,Y \in \Omega,\ X \equiv_{\pi} Y$.

Equivalently,

$\Omega/\pi \cong 1$

in the set-level case.

A join $\pi \vee \rho$ is degenerate if it collapses Ω or annihilates all nontrivial protected invariants.

A policy π is nontrivial if

$\exists X,Y \in \Omega$ such that not $X \equiv_\pi Y$.

Noncollapse is therefore a necessary condition for meaningful quotient formation.

4. The Admissibility Fibration

4.1 Indexed form

Let $\mathfrak{P}$ be a policy bicategory.

An admissibility fibration is a pseudofunctor

$\mathfrak{A} : \mathfrak{P}^{op} \to \mathrm{Cat}_\infty$.

Equivalently, by Grothendieck construction, it is a fibration

$p : \mathfrak{E} \to \mathfrak{P}$.

For each policy $\pi \in \mathfrak{P}$, the fiber

$\mathfrak{E}_\pi = p^{-1}(\pi)$

is the ∞-category of π-objects and π-certificates.

Objects of $\mathfrak{E}_\pi$ are policy-objects X_π.

Mapping objects in $\mathfrak{E}_\pi$ are certificate spaces:

$\mathrm{Map}_{\mathfrak{E}_\pi}(X_\pi, Y_\pi) = \mathrm{Cert}_\pi(X,Y)$.

Thus identity under π is represented by inhabitation of the mapping object:

$X \equiv_\pi Y \Leftrightarrow \mathrm{Map}_{\mathfrak{E}_\pi}(X_\pi, Y_\pi)$ is inhabited.

4.2 Total category

The total category $\mathfrak{E}$ has objects

(π, X_π)

where $\pi \in \mathfrak{P}$ and $X_\pi \in \mathfrak{E}_\pi$.

A morphism

$(\pi, X\pi) \to (\rho, Y\rho)$

consists of policy transport together with certificate transport.

In elementary notation, such a morphism is a pair

(α, c)

where

$\alpha : \pi \to \rho$

is a policy comparison and

$c : \alpha_{(!)}X\pi \to Y\rho$

is a certificate in the target fiber.

If transport is contravariant, one instead writes

$c : X\pi \to \alpha^*Y\rho$.

Thus morphisms in the total category simultaneously record change of admissibility jurisdiction and certified transformation within the resulting jurisdiction.

4.3 Reindexing

For $\alpha : \pi \to \rho$ in $\mathfrak{P}$, the fibration supplies reindexing functors

$\alpha^* : \mathfrak{E}\rho \to \mathfrak{E}\pi$.

When α is interpreted as refinement $\pi \leq \rho$, there is also a weakening functor

$W\pi\rho : \mathfrak{E}\pi \to \mathfrak{E}\rho$

whenever every π-certificate induces a ρ-certificate.

On mapping objects this gives

$\mathrm{Cert}\pi(X,Y) \to \mathrm{Cert}\rho(X,Y)$.

Hence

$X \equiv\pi Y \Rightarrow X \equiv\rho Y$.

Thus policy refinement is functorial weakening of identity judgment.

4.4 Fibers as identity regimes

Each fiber $\mathfrak{E}\pi$ is an identity regime.

If π is discrete, $\mathfrak{E}\pi$ is equivalent to a discrete category.

If π is an isomorphism policy, $\mathfrak{E}\pi$ is a groupoid of isomorphic presentations.

If π is a homotopy policy, $\mathfrak{E}\pi$ is an ∞-groupoid.

If π is a directed transformation policy, $\mathfrak{E}\pi$ is an ∞-category.

If π is observational, $\mathfrak{E}\pi$ is a quotient-like category of observationally indistinguishable presentations.

If π is gauge-theoretic, $\mathfrak{E}\pi$ is an orbit-stack-like fiber.

Therefore no single kind of quotient or identity type is primitive. The policy determines the fiber geometry.

4.5 Categories after policy

Let π be a policy satisfying:

$id_X \in Adm_\pi(X,X)$,

$f \in Adm_\pi(X,Y), g \in Adm_\pi(Y,Z) \Rightarrow g \circ f \in Adm_\pi(X,Z)$,

composition is associative,

certificate composition is coherent.

Then π determines an ordinary category $C\pi$ with

$Ob(C\pi) = \Omega\pi$,

$Hom_{C\pi}(X,Y) = Adm_\pi(X,Y)$.

If certificates are retained, $C\pi$ is enriched over certificate objects:

$Hom_{C\pi}(X,Y) = Cert_\pi(X,Y)$.

Thus category theory appears fiberwise after the admissibility policy has selected the arrows.

4.6 HoTT after policy

Let Ω be type-like. For $\pi \in \mathfrak{P}$ define the policy-indexed identity type

$Id_\pi(X,Y) := Cert_\pi(X,Y)$.

Then

$X \equiv_\pi Y \Leftrightarrow Id_\pi(X,Y)$ is inhabited.

A policy-indexed univalence principle has the form

$(X \simeq_\pi Y) \simeq \mathrm{Id}_\pi(X,Y)$,

where $\simeq_\pi$ denotes equivalence preserving Inv_π and satisfying Coh_π.

Thus HoTT appears internally to a fiber $\mathfrak{E}_\pi$. Admissibility Algebra governs which identity types are installed.

4.7 Higher certificates

Given $c,d \in \mathrm{Cert}_\pi(X,Y)$, define second-order certificates

$\mathrm{Cert}^2_\pi(c,d)$.

Given $h,k \in \mathrm{Cert}^2_\pi(c,d)$, define third-order certificates

$\mathrm{Cert}^3_\pi(h,k)$.

Continuing, one obtains a tower

$\mathrm{Cert}_\pi(X,Y)$,

$\mathrm{Cert}^2_\pi(c,d)$,

$\mathrm{Cert}^3_\pi(h,k)$,

...

This tower is the higher coherence structure of π-identity.

If every higher certificate is invertible, the fiber is ∞-groupoidal.

If higher certificates are directed, the fiber is ∞-categorical.

If higher certificates are truncated at level n, the fiber is n-categorical.

4.8 Internal language

The internal language of $\mathfrak{E}_\pi$ is a π-indexed type theory.

Judgments have the form

$\Gamma \vdash_\pi X \text{ type}$,

$\Gamma \vdash_\pi x : X$,

$\Gamma \vdash_\pi p : \mathrm{Id}_\pi(x,y)$.

Substitution is not global. It is valid only under contexts respecting π.

Transport along identity paths is replaced by certificate transport along admissible policy paths.

Thus the ordinary identity eliminator is refined into a policy-indexed eliminator.

5. Quotients as Fiberwise Colimits

5.1 Policy quotient

Let Ω be a presentation universe and π a policy.

The π-quotient is the universal object receiving a π-invariant map from Ω.

Write

$q_\pi : \Omega \to \Omega/\pi$.

A map $f : \Omega \to Z$ factors through q_π iff

$X \equiv_\pi Y \Rightarrow f(X) = f(Y)$

or, in proof-relevant form, iff f transports π-certificates to identity certificates in Z.

Thus Ω/π is universal among π-invariant targets.

5.2 Set-level quotient

If Cert_π is proposition-valued, symmetric, reflexive, and transitive, then $\equiv_\pi$ is an equivalence relation and

$\Omega/\pi = \{\mathrm{Orb}_\pi(X) : X \in \Omega\}$.

The quotient map is

$q_\pi(X) = \mathrm{Orb}_\pi(X)$.

This is the ordinary set quotient.

5.3 Groupoidal quotient

If $\mathrm{Cert}_\pi(X,Y)$ is a groupoid of witnesses, then Ω/π is not adequately represented by equivalence classes.

The correct quotient is a homotopy quotient.

The orbit of X is the full subgroupoid generated by all Y with $\mathrm{Cert}_\pi(X,Y)$ inhabited and all higher certificates among witnesses.

Thus quotienting preserves witness geometry.

5.4 ∞-categorical quotient

If Cert_π is ∞-groupoid-valued, Ω/π is an ∞-groupoid quotient.

If Certπ is directed, Ω/π is an ∞-categorical localization or directed quotient.

If Certπ is partially defined, Ω/π is a partial localization.

If Certπ is context-indexed, Ω/π is descent data in the admissibility fibration.

Thus the notation Ω/π is schematic. Its realization depends on the certificate type.

5.5 Universal property

Let $\mathcal{C}$ be the ambient category of presentations and admissible maps.

A π-quotient is an object Ω/π equipped with qπ : Ω → Ω/π such that for any object Z and any π-invariant map f : Ω → Z, there exists a unique induced map $\bar{f}$: Ω/π → Z satisfying

$f = \bar{f} \circ q\pi$.

In homotopical form, uniqueness is replaced by contractibility of the space of such factorizations.

In ∞-categorical form,

Map(Ω/π,Z) ≃ Mapπ-inv(Ω,Z).

5.6 Homotopy-coherent quotient

For proof-relevant policies, π-invariance of f requires more than equal values on equivalent presentations.

For each c ∈ Certπ(X,Y), one requires a specified path

fc : f(X) → f(Y).

For each higher certificate h : c ⇒ d, one requires coherence

fh : fc ⇒ fd.

Thus f is π-invariant iff it maps the full certificate tower of π into the identity tower of Z.

5.7 Quotient collapse

A quotient Ω/π collapses if Ω/π is terminal or contractible.

Set-level collapse:

|Ω/π| = 1.

Homotopy-level collapse:

Ω/π is contractible.

Categorical collapse:

Ω/π is equivalent to the terminal category.

Policy collapse is the quotient-theoretic expression of excessive admissibility.

A policy that admits too many transformations destroys distinguishability.

5.8 Nontriviality

A quotient Ω/π is nontrivial if $q\pi$ is neither injective nor terminal.

Thus π identifies some distinct presentations but not all presentations.

This is the useful region of admissibility:

$id\Omega < \pi <$ collapse.

In this interval, π performs genuine abstraction.

5.9 Quotient comparison

If $\pi \leq \rho$, then there is a canonical quotient comparison

$\Omega/\pi \to \Omega/\rho$.

Indeed, π-identity implies ρ-identity. Therefore every ρ-class is a union of π-classes.

The quotient map factors:

$\Omega \to \Omega/\pi \to \Omega/\rho$.

Thus policy refinement induces quotient coarsening.

5.10 Quotient functoriality

For fixed Ω, the assignment

$\pi \mapsto \Omega/\pi$

is order-reversing with respect to strictness and coarsening with respect to admissibility.

If $\pi \leq \rho$, then Ω/π is finer than Ω/ρ.

Equivalently,

stricter policy → smaller orbits → finer quotient,

weaker policy → larger orbits → coarser quotient.

5.11 Stabilizers

Let $X \in \Omega$. The stabilizer

$\mathrm{Stab}_\pi(X)$

acts on the presentation X by admissible self-transformations preserving $\mathrm{Inv}_\pi(X)$.

If π is groupoidal, $\mathrm{Stab}_\pi(X)$ is a group or ∞-group.

If π is categorical, $\mathrm{Stab}_\pi(X)$ is a monoid or endomorphism ∞-category.

If π is directed, $\mathrm{Stab}_\pi(X)$ may encode irreversible self-maintenance.

The orbit-stabilizer relation generalizes:

$\mathrm{Orb}_\pi(X)$ is generated by admissible transformations from X,

$\mathrm{Stab}_\pi(X)$ is the internal symmetry of X under π.

5.12 Orbit-stack form

When certificates are nontrivial, the quotient should be understood as an orbit stack:

$\Omega//\pi$.

This retains automorphism and stabilizer data lost in coarse quotienting.

Thus the three levels are:

Ω/π : coarse quotient,

$\Omega//\pi$: quotient retaining stabilizers,

$\Omega/h\pi$: homotopy-coherent quotient.

The correct level is determined by Cert_π.

6. Scoped Substitution

6.1 Failure of global substitution

Classical equality satisfies Leibniz substitution:

$X = Y \Rightarrow C[X] = C[Y]$

for all contexts C[-].

Policy-relative identity does not.

From

$X \equiv_\pi Y$

one cannot infer

$C[X] = C[Y]$

for arbitrary C.

The context C must respect the policy π.

6.2 π-respecting context

Let C[-] be a context, construction, predicate, functor, observable, theory, or test.

C respects π if for all X,Y,

$X \equiv_\pi Y \Rightarrow C[X] \equiv_\sigma C[Y]$

for some target policy σ.

Equivalently, C transports π-certificates into σ-certificates:

$\mathrm{Cert}_\pi(X,Y) \to \mathrm{Cert}_\sigma(C[X],C[Y])$.

If σ is ordinary equality, then C is strictly π-invariant.

If σ is another admissibility policy, then C is policy-functorial.

6.3 Scoped substitution principle

If $X \equiv_\pi Y$ and C respects π, then

$C[X] \equiv_\sigma C[Y]$.

This is the scoped substitution principle.

It replaces the unrestricted Leibniz rule by a typed, indexed, policy-sensitive rule.

6.4 Policy functors

A map $F : \mathfrak{E}_\pi \to \mathfrak{E}_\sigma$ is policy-functorial if it sends π-certificates to σ-certificates.

That is, for each X,Y there is a map

$\mathrm{Cert}_\pi(X,Y) \to \mathrm{Cert}_\sigma(FX,FY)$

compatible with higher certificates.

Such an F preserves identity only relative to the source and target policies.

Thus functoriality itself becomes policy-indexed.

6.5 Substitution theorem

Theorem 6.5.1. Let C[-] respect π with target policy σ. If X ≡π Y, then C[X] ≡σ C[Y].

Proof. Since X ≡π Y, Certπ(X,Y) is inhabited. Since C respects π, there is a certificate transport map Certπ(X,Y) → Certσ(C[X],C[Y]). Hence Certσ(C[X],C[Y]) is inhabited. Therefore C[X] ≡σ C[Y].

6.6 Nonconverse

The converse generally fails.

C[X] ≡σ C[Y] does not imply X ≡π Y.

A context may forget distinctions. Thus policy-respecting maps are generally quotienting, not reflecting.

If C both preserves and reflects certificates, then C is policy-faithful.

6.7 Policy-faithfulness

A policy functor F : 𝔈π → 𝔈σ is faithful with respect to certificates if

Certπ(X,Y) → Certσ(FX,FY)

is injective, fully faithful, or an embedding at the relevant categorical level.

F reflects identity if

FX ≡σ FY ⇒ X ≡π Y.

If F preserves and reflects identity, then F is policy-conservative.

6.8 Substitution and invariants

Let Invπ be the protected invariant of π.

A context C respects π only if C depends on X through Invπ(X), or through data functorially determined by Invπ(X).

Thus scoped substitution can be expressed as:

C[X] = C[Y] whenever Invπ(X) ≅ Invπ(Y) by certified admissible transport.

This gives the invariant-theoretic form of scoped substitution.

6.9 Logical form

A policy-indexed logic has judgments

Γ ⊢π X ≡ Y.

Elimination has the restricted form:

$\Gamma \vdash_\pi p : X \equiv Y$,

C respects π,

therefore $\Gamma \vdash_\sigma C[X] \equiv C[Y]$.

There is no global identity eliminator independent of policy.

6.10 Consequence

Equivalence never licenses substitution by itself.

Only certified admissibility under a stated policy licenses substitution in compatible contexts.

This is the central logical correction made by Admissibility Algebra.

7. Policy-Indexed Dynamics

7.1 Dynamical presentation space

Let S be a presentation space equipped with either a discrete-time evolution

$\Phi : S \to S$

or a continuous-time flow

$\Phi t : S \to S,\ t \geq 0$.

Let π be an admissibility policy on S.

The dynamical question is not whether Φ preserves pointwise equality, but whether Φ preserves π-identity.

7.2 π-invariant dynamics

The dynamics Φ is π-invariant if

$X \equiv_\pi Y \Rightarrow \Phi(X) \equiv_\pi \Phi(Y)$.

Equivalently, Φ induces a map on the quotient:

$\Phi\pi : S/\pi \to S/\pi$

such that

$q\pi \circ \Phi = \Phi\pi \circ q\pi$

or, in proof-relevant form,

$q\pi \circ \Phi \simeq \Phi\pi \circ q\pi$

by coherent certificate.

Thus π-invariant dynamics descends to the π-quotient.

7.3 π-equivariant dynamics

Let π be represented by admissible transformations Admπ. Φ is π-equivariant if for each $t \in \mathrm{Adm}\pi(X,Y)$, there exists an admissible transformation

$\Phi(t) \in \mathrm{Adm}\pi(\Phi X, \Phi Y)$

preserving certificates and coherence.

At certificate level, this requires maps

$\mathrm{Cert}\pi(X,Y) \to \mathrm{Cert}\pi(\Phi X,\Phi Y)$

compatible with composition.

Thus π-equivariance is functoriality of Φ inside the fiber $\mathfrak{C}\pi$.

7.4 π-stability

Let Eπ be a policy-object in S.

Let Orbπ(E) be its π-orbit.

A dynamics Φ is stable relative to Eπ if trajectories beginning near Orbπ(E) remain near Orbπ(E) in a π-sensitive topology, metric, pseudometric, divergence, or certificate-distance structure.

The stable set is not a fixed point.

It is the π-orbit:

Orbπ(E).

Thus classical equilibrium stability is the special case where π is discrete and Orbπ(E) = {E}.

7.5 Policy pseudometric

A policy pseudometric dπ on S satisfies

$d\pi(X,Y) = 0 \Leftrightarrow X \equiv\pi Y$

or, more generally,

$d\pi(X,Y) = 0 \Leftrightarrow$ X and Y lie in the same π-orbit.

Such dπ descends to a metric on S/π when the quotient is sufficiently separated.

A policy pseudometric may be constructed from invariant defect:

$d_\pi(X,Y) = \delta(\mathrm{Inv}_\pi(X), \mathrm{Inv}_\pi(Y))$,

from certificate obstruction,

$d_\pi(X,Y) = \inf \mathrm{obstruction}_\pi(c)$,

or from Lyapunov tension.

7.6 Basins

The π-basin of E is

$B_\pi(E) = \{X \in S : \lim t\to\infty\ d_\pi(\Phi_t X, \mathrm{Orb}_\pi(E)) = 0\}$

in the asymptotic case.

The weak π-basin is

$B_\pi^{\mathrm{weak}}(E) = \{X \in S : \sup_{t\geq 0} d_\pi(\Phi_t X, \mathrm{Orb}_\pi(E)) < \infty\}$.

The finite-threshold π-basin is

$B_\pi^{\theta}(E) = \{X \in S : V_\pi(\Phi_t X) \leq \theta \text{ for all } t \geq 0\}$.

Thus basin membership is policy-relative.

7.7 Identity persistence

A trajectory X(t) persists as E_π on interval I if

$X(t) \in \mathrm{Orb}_\pi(E)$

for all $t \in I$,

or more weakly if

$V_\pi(X(t)) \leq \theta_\pi$

for all $t \in I$.

Identity persistence is therefore bounded policy-tension.

7.8 Identity failure

A trajectory undergoes π-identity failure at time τ if

$X(\tau) \notin B_\pi(E)$

or

$V\pi(X(\tau)) > \theta\pi$.

In exact form, failure occurs when $Cert\pi(E,X(\tau))$ is empty.

In approximate form, failure occurs when certificate obstruction exceeds tolerance.

7.9 Multiple policies

Let $\pi,\rho \in \Pi$.

A single trajectory may be stable under π and unstable under ρ.

Thus stability is not absolute.

One may have

$\Delta V\pi \leq 0$

and

$\Delta V\rho > 0$

on the same trajectory.

This expresses the incompatibility of identity regimes.

7.10 Policy spectrum

Given a family Π of policies, define the stability spectrum of X under Φ as

$\Sigma\Phi(X) = \{\pi \in \Pi : X \text{ is } \pi\text{-stable under } \Phi\}$.

The instability spectrum is

$\Lambda\Phi(X) = \{\pi \in \Pi : X \text{ is } \pi\text{-unstable under } \Phi\}$.

The boundary between $\Sigma\Phi(X)$ and $\Lambda\Phi(X)$ is the policy-critical surface.

8. Lyapunov Enrichment

8.1 Policy-relative Lyapunov function

Let S be a dynamical presentation space and π a policy.

A π-Lyapunov function for policy-object $E\pi$ is a map

$V\pi : S \to \mathbb{R}\geq 0$

such that

$V\pi(X) = 0 \Leftrightarrow X \in Orb\pi(E)$.

For discrete time,

$\Delta V_\pi(X) = V_\pi(\Phi X) - V_\pi(X)$.

For continuous time,

$\dot{D}V_\pi(X) = d/dt\ V_\pi(\Phi_t X)|_{t=0}$.

The stability condition is

$\Delta V_\pi(X) \leq 0$

or

$\dot{D}V_\pi(X) \leq 0$

on the relevant basin.

8.2 Difference from classical Lyapunov theory

Classical Lyapunov theory usually takes the zero set to be an equilibrium point, invariant set, or attractor.

Admissibility Lyapunov theory takes the zero set to be a policy-orbit:

$V_\pi^{-1}(0) = Orb_\pi(E)$.

Thus the object stabilized by the dynamics is not a microstate but an admissibility class.

8.3 Tension interpretation

V_π measures policy-tension.

$V_\pi(X) = 0$ means no π-defect.

$V_\pi(X) > 0$ means deviation from π-identity.

Increasing V_π means identity strain.

Decreasing V_π means return toward the π-orbit.

Threshold crossing

$V_\pi(X) > \theta_\pi$

marks rupture of π-identity.

8.4 Certificate-distance form

Let $Obs_\pi(X,Y)$ be an obstruction object measuring failure of $Cert_\pi(X,Y)$ to be inhabited.

Define

$V_\pi(X) = \mathrm{Obs}_\pi(E,X)$

or

$V_\pi(X) = \inf_c \mathrm{obstruction}_\pi(c)$

over candidate certificates c.

Then V_π is zero precisely when a valid π-certificate exists.

This converts proof failure into dynamical tension.

8.5 Invariant-defect form

Let Inv_π be valued in a metric or normed space.

Define

$V_\pi(X) = ||\mathrm{Inv}_\pi(X) - \mathrm{Inv}_\pi(E)||^2$.

More generally, if $\mathrm{Inv}_\pi(X)$ and $\mathrm{Inv}_\pi(E)$ are structured objects, V_π is a defect functional on invariant comparison.

Then V_π measures violation of protected invariants.

8.6 Quotient-distance form

Let $q_\pi : S \to S/\pi$.

If S/π has metric $\bar{d}_\pi$, define

$V_\pi(X) = \bar{d}_\pi(q_\pi(X), q_\pi(E))^2$.

This is appropriate when policy identity has already been quotiented.

In this form, V_π is ordinary Lyapunov theory on the quotient.

8.7 Orbit-distance form

If S has a metric d and $\mathrm{Orb}_\pi(E)$ is a subset, define

$V_\pi(X) = \inf_{Y \in \mathrm{Orb}_\pi(E)} d(X,Y)^2$.

This is the nearest-orbit Lyapunov function.

It is valid when π-orbits are geometrically embedded in S.

8.8 Stability theorem

Theorem 8.8.1. Let $V\pi : S \to \mathbb{R}\geq 0$ satisfy:

$V\pi(X) = 0$ iff $X \in Orb\pi(E)$,

$V\pi$ is continuous,

$\Delta V\pi(X) \leq 0$ on a neighborhood U of $Orb\pi(E)$.

Then $Orb\pi(E)$ is Lyapunov-stable in U.

Proof. This is the standard Lyapunov argument applied to the invariant set $Orb\pi(E)$. Since $V\pi$ is nonincreasing along trajectories and vanishes exactly on the orbit, sufficiently small sublevel neighborhoods remain forward-invariant. Hence trajectories starting sufficiently near $Orb\pi(E)$ remain near $Orb\pi(E)$.

8.9 Asymptotic theorem

Theorem 8.9.1. If, in addition,

$\Delta V\pi(X) < 0$

for $X \notin Orb\pi(E)$ in U, then $Orb\pi(E)$ is asymptotically π-stable, subject to the usual compactness or LaSalle-type hypotheses.

Proof. Apply the invariant-set form of Lyapunov-LaSalle theory to the zero set $Orb\pi(E)$.

8.10 LaSalle form

Let

$M\pi = \{X \in U : \Delta V\pi(X)=0\}$.

If the largest invariant subset of $M\pi$ is $Orb\pi(E)$, then every trajectory in U approaches $Orb\pi(E)$.

Thus asymptotic π-identity need not require strict descent everywhere.

It suffices that the only invariant no-descent set is the policy-orbit.

8.11 Multiple Lyapunov functions

For a family of policies Π, one obtains a family

$\{V\pi\}\pi\in\Pi$.

A system may admit simultaneous stability under a compatible family $\Pi_o \subseteq \Pi$ if

$\Delta V\pi \leq 0$ for all $\pi \in \Pi_o$.

If $V\pi$ and $V\rho$ impose incompatible descent requirements, no common basin may exist.

Thus policy compatibility becomes a multi-Lyapunov feasibility problem.

8.12 Policy refinement and Lyapunov monotonicity

Suppose $\pi \leq \rho$.

Then $Orb\pi(E) \subseteq Orb\rho(E)$.

A natural compatibility condition is

$V\rho(X) \leq V\pi(X)$

after normalization.

Strict policies measure more defects.

Weak policies ignore more defects.

Thus refinement should induce monotone tension:

$\pi \leq \rho \Rightarrow V\rho \leq V\pi$.

This need not hold automatically; it is an additional coherence condition for Lyapunov-enriched admissibility systems.

8.13 Lyapunov-enriched admissibility fibration

A Lyapunov-enriched admissibility fibration is a fibration

$p : \mathfrak{E} \to \mathfrak{P}$

together with, for each $\pi \in \mathfrak{P}$, a functional

$V\pi : Obj(\mathfrak{E}\pi) \to \mathbb{R}_{\geq 0}$

or more generally

$V\pi : \mathfrak{E}\pi \to \mathcal{E}$

for an ordered enrichment category $\mathcal{E}$,

such that policy transport preserves or monotonically relates tension.

For $\alpha : \pi \to \rho$, require either

$V\rho(W\pi\rho X) \leq V\pi(X)$

for weakening,

or

$V\pi(\alpha^*Y) \geq V\rho(Y)$

for restriction.

8.14 Stability as enriched descent

In this setting, dynamics is an endofunctor or flow

$\Phi : \mathfrak{E}\pi \to \mathfrak{E}\pi$.

π-stability is the enriched descent condition

$V\pi(\Phi X) \leq V\pi(X)$.

Thus Lyapunov theory becomes order-enriched dynamics in a policy fiber.

9. Stable Quotient Formation

9.1 Microstate space

Let M be a microstate presentation space with dynamics

$\Phi : M \to M$.

Let π be a policy on M.

Let

$q\pi : M \to M/\pi$

be the admissibility quotient.

A macro-object is an object $E \in M/\pi$.

A representative microstate $x \in M$ belongs to E if

$q\pi(x)=E$.

9.2 Descent of dynamics

The dynamics Φ descends to M/π if there exists $\Phi\pi : M/\pi \to M/\pi$ such that

$q\pi \circ \Phi = \Phi\pi \circ q\pi$.

In proof-relevant form, require a coherent certificate

$q\pi \circ \Phi \simeq \Phi\pi \circ q\pi$.

This condition means that micro-dynamics respects π-identity.

9.3 Stable quotient

A quotient M/π is stable on basin $B \subseteq M$ if:

B is forward-invariant under Φ,

Φ descends to $\Phi\pi$ on $q\pi(B)$,

there exists $V\pi$ with zero set $q\pi^{-1}(E)$,

$V\pi(\Phi x) \leq V\pi(x)$ for $x \in B$.

Thus stability belongs to the quotient object, not to one representative.

9.4 Nontrivial quotient

A stable quotient is nontrivial if

$q\pi$ is not injective

and

$q\pi$ is not terminal.

Equivalently, π identifies some microstates but not all microstates.

This excludes both discrete identity and total collapse.

9.5 Definition of emergence

An emergent object $E\pi$ is a nontrivial quotient object $E \in M/\pi$ satisfying:

1. $q\pi$ is nontrivial.
2. E has inhabited certificate structure.
3. $Inv\pi$ is preserved on E.
4. Φ descends to E up to certificate.
5. There exists a basin $B\pi(E)$.
6. There exists $V\pi$ such that $V\pi^{-1}(0)=q\pi^{-1}(E)$.
7. $V\pi$ is nonincreasing on $B\pi(E)$.

Then $E\pi$ is a Lyapunov-stable admissibility quotient.

9.6 Emergence theorem

Theorem 9.6.1. Let M be a dynamical presentation space, π a nontrivial policy, and $q\pi : M \to M/\pi$ a quotient. Suppose there exists $E \in M/\pi$ and $V\pi : M \to \mathbb{R}\geq 0$ such that:

$V\pi(x)=0$ iff $q\pi(x)=E$,

$V\pi(\Phi x) \leq V\pi(x)$ on a neighborhood B of $q\pi^{-1}(E)$,

B is forward-invariant,

and Φ descends to M/π on B.

Then E is a stable emergent object under π.

Proof. Since $q\pi$ is nontrivial, E is not reducible to a single presentation and not collapsed with all presentations. Since $V\pi$ vanishes exactly on $q\pi^{-1}(E)$ and is nonincreasing on B, $q\pi^{-1}(E)$ is Lyapunov-stable. Since Φ descends to M/π, the quotient object has induced dynamics independent of representative. Hence E is a dynamically stable quotient object.

9.7 Strong emergence condition

The emergent quotient is strong if the induced dynamics $\Phi\pi$ on M/π is not equivalent to the restriction of Φ to any single representative subspace.

Formally, there is no section $s : M/\pi \to M$ such that

$q\pi \circ s = id$

and

$s \circ \Phi\pi = \Phi \circ s$

up to the same policy.

If no such representative dynamics exists, the quotient has autonomous macrostructure.

9.8 Weak emergence condition

The emergent quotient is weak if such a section exists locally or approximately.

Then the macro-object is stable but representable by chosen microstates.

Weak emergence is stable quotienting with representative dependence.

Strong emergence is stable quotienting without faithful representative reduction.

9.9 Emergence and coarse-graining

Every admissibility quotient is a coarse-graining in the formal sense that qπ identifies lower-level distinctions.

Not every coarse-graining is emergence.

A coarse-graining becomes emergence only when the quotient is:

nontrivial,

certified,

invariant-preserving,

dynamically stable,

and supports induced transformations.

Thus emergence is certified stable coarse-graining.

9.10 Emergence and phase transition

Let $\pi\lambda$ be a family of policies or $V\lambda$ a family of tensions depending on parameter λ.

A phase transition occurs at λc if the stable quotient structure changes discontinuously:

$M/\pi\lambda^{-} \not\simeq M/\pi\lambda^{+}$

or the basin structure changes:

$B\pi\lambda^{-}(E) \neq B\pi\lambda^{+}(E)$.

The order parameter is any invariant detecting the quotient change.

Emergence occurs when a previously unstable or nonexistent quotient becomes stable.

9.11 Rupture/emergence duality

Let π and ρ be policies.

A trajectory x_n undergoes rupture under π and emergence under ρ if

$V\pi(x_n)$ increases beyond $\theta\pi$,

while

$V\rho(x_n)$ decreases toward 0.

Thus the same dynamical event may be identity failure relative to π and identity formation relative to ρ.

This is cross-policy basin transfer.

9.12 General formula

Emergence may be summarized as:

microstates M,

policy π,

quotient $q\pi : M \to M/\pi$,

dynamics Φ,

Lyapunov tension Vπ,

stable macro-object $E \in M/\pi$.

Then:

E emerges iff Ω/π is nontrivial and Vπ stabilizes $q\pi^{-1}(E)$.

In compressed notation:

Emergence = Lyapunov-stable admissibility quotient formation.

10. Finite Admissibility Object: The Genetic Code

10.1 Alphabet and codon space

Let

$\mathcal{A} = \{G,A,C,U\}$.

Let

$\mathcal{C} = \mathcal{A}^3$.

Elements $c \in \mathcal{C}$ are written

$c = c_1c_2c_3$

with $c_i \in \mathcal{A}$.

Thus

$|\mathcal{C}| = 4^3 = 64$.

Let

$\mathcal{R} = AA_{20} \cup \{X\}$

where AA_{20} denotes the twenty standard amino-acid labels and X denotes termination.

Let

$\phi : \mathcal{C} \to \mathcal{R}$

be the standard translation map.

The pair

$(\mathcal{C},\phi)$

is a finite symbolic translation system.

10.2 Translation policy

Define the translation policy $\pi\phi$ by

$c \equiv_\phi d \Leftrightarrow \phi(c)=\phi(d)$.

Then $\equiv_\phi$ is an equivalence relation on $\mathcal{C}$.

The translation quotient is

$\mathcal{C}/\pi\phi \cong \mathcal{R}$.

Thus the ordinary genetic-code table is the quotient representation of codon space under translated identity.

The fiber over $r \in \mathcal{R}$ is

$\mathcal{C}r = \phi^{-1}(r)$.

Synonymous codons are elements of the same fiber.

Thus degeneracy is stabilizer structure relative to $\pi\phi$.

10.3 Policy-object form

The standard genetic code determines a finite policy-object

$\mathcal{G}std = (\mathcal{C}, \phi, D, Q, o)$

where:

$\mathcal{C} = \mathcal{A}^3$ is codon presentation space,

$\phi : \mathcal{C} \to \mathcal{R}$ is translated identity,

$D \in \{0,1\}^{64x12}$ is the nucleotide-position incidence matrix,

$Q : \mathcal{R} \to \mathbb{R}^m$ is a property assignment,

o is an ordering of $\mathcal{A}$.

Each component induces a policy or projection:

ϕ induces translation quotient,

D induces additive nucleotide-position projection,

Q induces physicochemical field pullback,

o induces ordered-cube topology.

Thus $\mathcal{G}$std is not merely a table. It is a finite admissibility object with multiple compatible policies.

10.4 Incidence matrix

Index the twelve columns of D by pairs

$(p,s) \in \{1,2,3\} \times \mathcal{A}$.

Index the sixty-four rows by codons $c \in \mathcal{C}$.

Define

$D^c,(p,s) = 1$ iff $c_p = s$,

$D^c,(p,s) = 0$ otherwise.

Each row has exactly three ones.

For each position p, the four columns

$D_(p,G_), D_(p,A_), D_(p,C_), D_(p,U_)$

sum to the all-ones vector $\mathbb{1} \in \mathbb{R}^{64}$.

Hence

$D_(1,G_) + D_(1,A_) + D_(1,C_) + D_(1,U_)$

=

$D_(2,G_) + D_(2,A_) + D_(2,C_) + D_(2,U_)$

=

$D_(3,G_) + D_(3,A_) + D_(3,C_) + D_(3,U_)$

=

$\mathbb{1}$.

Therefore the twelve columns satisfy two independent linear dependencies.

No further dependencies occur.

Hence

rank(D)=10.

10.5 Exact pseudoinverse

Let J be the 12 × 64 all-ones matrix.

For the full codon incidence matrix D,

$D^{+} = 1/16\ D^{T} - 1/96\ J$.

The associated projection is

$PD = DD^{+}$.

Thus

PD : $\mathbb{R}^{64} \rightarrow$ Col(D)

is the orthogonal projection onto the rank-10 additive nucleotide-position subspace.

A codon-level scalar field $f \in \mathbb{R}^{64}$ has additive approximation

fadd = PD f.

The residual is

r = f − PD f.

The residual measures structure not captured by the additive nucleotide-position policy.

10.6 Property pullback

Let

Q : $\mathcal{R} \rightarrow \mathbb{R}^{m}$.

Define the codon-level property field

F : $\mathcal{C} \rightarrow \mathbb{R}^{m}$

by

F = Q ∘ ϕ.

Equivalently, F is the pullback of Q along the quotient map ϕ.

The additive projection is

Fadd = PD F

componentwise.

The residual

R = F − Fadd

measures failure of the rank-10 additive policy.

Thus D defines a projection policy πD, and R is the corresponding policy-defect.

10.7 Translation quotient and projection do not commute trivially

There are two structures:

qϕ : $\mathcal{C} \to \mathcal{C}/\pi\phi \cong \mathcal{R}$,

PD : $\mathbb{R}^{64} \to$ Col(D).

The quotient qϕ collapses codons by translated label.

The projection PD collapses codon fields by additive nucleotide-position structure.

These are different admissibility reductions.

Their interaction is nontrivial because synonymy fibers need not align with additive-position subspaces.

Thus the genetic code carries at least two simultaneous policies:

πϕ = translated identity,

πD = additive nucleotide-position identity.

The combined policy

πϕ ∧ πD

is stricter than either.

The joined policy

πϕ ∨ πD

requires coherence between symbolic translation and additive physicochemical projection.

10.8 Ordered cube

Let o be a total ordering of $\mathcal{A}$.

An ordering o identifies $\mathcal{A}$ with {1,2,3,4}. Hence $\mathcal{C}$ becomes the lattice cube

$\{1,2,3,4\}^3$.

Two codons c,d are nearest neighbors under o if their coordinates differ by one in exactly one position and agree in the other two.

The number of nearest-neighbor edges is

$3 \cdot (4-1) \cdot 4^2 = 144$.

Thus each ordering o induces a graph

$\Gamma o = (\mathcal{C}, Eo)$

with $|\mathcal{C}|=64$ and $|Eo|=144$.

10.9 Pushforward multigraph

Push Γo forward along ϕ.

The vertex set becomes $\mathcal{R}$.

Each edge c—d ∈ Eo gives:

a self-loop at r if ϕ(c)=ϕ(d)=r,

an edge r—s if ϕ(c)=r and ϕ(d)=s with r≠s.

Thus $\phi_{(*)}\Gamma o$ is a multigraph on $\mathcal{R}$.

Self-loops are nearest-neighbor transformations preserving translated identity.

Non-loop edges are nearest-neighbor transformations crossing translation classes.

Therefore self-loops are local zero-tension edges under πϕ.

10.10 Discrete translation tension

Define edge tension

τϕ(c,d) = 0 if ϕ(c)=ϕ(d),

τϕ(c,d) = 1 if ϕ(c)≠ϕ(d).

For $Q : \mathcal{R} \to \mathbb{R}^m$, define property tension

$\tau Q(c,d) = ||Q(\phi(c)) - Q(\phi(d))||$.

For a path $\gamma = (c_0,c_1,...,c_n)$, define

$T\phi(\gamma) = \Sigma_i \tau\phi(c_i,c_{i+1})$,

$TQ(\gamma) = \Sigma_i \tau Q(c_i,c_{i+1})$.

Then $T\phi(\gamma)=0$ iff γ remains inside one synonymy fiber.

Thus $\pi\phi$-stable local motion corresponds to zero-tension paths in Γo.

10.11 Alphabet-order optimization

Let $L(o)$ be the number of synonymous nearest-neighbor edges in Γo:

$L(o) = |\{c—d \in Eo : \phi(c)=\phi(d)\}|$.

Equivalently, $L(o)$ is the number of self-loop edges in $\phi_{(*)}\Gamma o$, counted with multiplicity.

The alphabet-order problem is:

maximize $L(o)$ over orderings o of $\mathcal{A}$.

For anisotropic ordering, choose a triple

$(o_1,o_2,o_3) \in \mathrm{Sym}(\mathcal{A})^3$.

Then the graph has nearest-neighbor structure determined independently on the three axes.

The search space has size

$24^3 = 13{,}824$.

The objective becomes

$L(o_1,o_2,o_3)$.

This is a finite combinatorial admissibility problem.

10.12 Genetic-code object as finite test case

The finite admissibility object $\mathcal{G}$std supplies:

a finite presentation space $\mathcal{C}$,

a symbolic quotient $\pi\phi$,

a rank-10 projection policy πD,

a graph-topological policy π_o,

a property pullback $Q \circ \phi$,

a tension functional τ_ϕ or τ_Q,

a multigraph quotient $\phi_{(*)}\Gamma_o$,

a finite optimization problem $L(o)$.

Thus the genetic code is a fully explicit model of:

policy-indexed identity,

certified quotienting,

projection under invariant policy,

local stability as zero-tension adjacency,

and policy-dependent topology.

11. Learned and Approximate Admissibility Systems

11.1 Approximate identity

Many systems do not admit exact certificates.

Let Ω be a presentation space.

Let π be a policy.

Instead of a proposition-valued certificate object $\mathrm{Cert}_\pi(X,Y)$, suppose there is a score

$s_\pi(X,Y) \in \mathbb{R}$

or a defect

$\delta_\pi(X,Y) \in \mathbb{R}_{\geq 0}$.

Define approximate admissibility by threshold:

$X \equiv_{\pi,\varepsilon} Y \Leftrightarrow \delta_\pi(X,Y) \leq \varepsilon$.

Exact identity is recovered at $\varepsilon=0$ when δ_π is certificate-complete.

Thus approximate admissibility is a filtered family of policies

$\pi_o \leq \pi_\varepsilon \leq \pi_{\varepsilon'}$

for $\varepsilon \leq \varepsilon'$.

As tolerance increases, orbits grow and quotients coarsen.

11.2 Filtered policy system

Let $\varepsilon \in \mathbb{R}_{\geq 0}$.

A filtered admissibility system is a monotone family

$\{\pi_\varepsilon\}_{\varepsilon \geq 0}$

such that

$\varepsilon \leq \varepsilon' \Rightarrow \pi_\varepsilon \leq \pi_{\varepsilon'}$

in the strictness order.

Equivalently,

$\mathrm{Cert}_{\pi_\varepsilon}(X,Y) \rightarrow \mathrm{Cert}_{\pi_{\varepsilon'}}(X,Y)$.

The associated quotients form a persistence diagram:

$\Omega/\pi_0 \rightarrow \Omega/\pi_\varepsilon \rightarrow \Omega/\pi_{\varepsilon'} \rightarrow \ldots$

Thus approximate identity produces persistent admissibility topology.

11.3 Probabilistic certificates

Let $\mathrm{Cert}_\pi(X,Y)$ be replaced by a probability

$P_\pi(X,Y) \in [0,1]$.

Define

$X \equiv_{\pi,p} Y \Leftrightarrow P_\pi(X,Y) \geq p$.

Alternatively, define tension

$V_\pi(X,Y) = -\log P_\pi(X,Y)$.

Then

$X \equiv_{\pi,p} Y \Leftrightarrow V_\pi(X,Y) \leq -\log p$.

Thus probabilistic admissibility is equivalent to thresholded tension.

11.4 Learned admissibility object

Let Ω be a symbolic presentation space.

Let θ be parameters of a learned transformation system.

Let

$f_\theta : \Omega \times K \to \mathrm{Dist}(\Omega)$

assign to a presentation and context a distribution over continuations.

A learned admissibility object is

$A_\theta = (\Omega, T_\theta, \Pi_\theta, \mathrm{Adm}_\theta, \mathrm{Inv}_\theta, \mathrm{Cert}_\theta, V_\theta)$

where:

T_θ is the class of model-generated transformations,

Π_θ is the class of task or context policies,

$\mathrm{Adm}_{\theta,\pi}(X,Y)$ is learned or thresholded admissibility,

$\mathrm{Inv}_{\theta,\pi}$ is the task-preserved invariant,

$\mathrm{Cert}_{\theta,\pi}$ is a validation, proof, execution, agreement, type-check, or external witness,

$V_{\theta,\pi}$ is loss, contradiction, distance, or policy-tension.

Then

$X \equiv_{\theta,\pi} Y$

means that the model treats X and Y as equivalent under π with sufficient certificate or sufficiently low tension.

11.5 Embedding neighborhoods

Suppose $e_\theta : \Omega \to \mathbb{R}^n$ is an embedding.

Let d be a metric or divergence on $\mathbb{R}^n$.

Define

$\delta_{\theta,\pi}(X,Y) = d_\pi(e_\theta(X), e_\theta(Y))$

where d_π may depend on task policy π.

Then

$X \equiv_{\theta,\pi,\varepsilon} Y \Leftrightarrow \delta_{\theta,\pi}(X,Y) \leq \varepsilon.$

The quotient $\Omega/\pi_{\theta,\varepsilon}$ is an approximate semantic quotient.

Embedding space therefore supplies a geometric representation of learned policy-orbits.

11.6 Task-conditioned policies

A single pair X,Y may be equivalent under one task and inequivalent under another.

Let $k \in K$ be a task context.

Let πk be the induced policy.

Then

$X \equiv_{\pi k} Y$

does not imply

$X \equiv_{\pi l} Y$

for $k \neq l$.

Thus learned systems naturally instantiate a context-indexed admissibility fibration

$p : \mathfrak{E}_\theta \to K$.

Fibers correspond to task-conditioned identity regimes.

11.7 Generation as admissible continuation

Let X be a prompt or initial presentation.

Let Y be a continuation.

A generated continuation Y is admissible under π if

$Y \in \mathrm{Adm}_{\theta,\pi}(X)$

and preserves the requested invariant Inv_π.

The invariant may be answerhood, meaning, tone, syntax, executable behavior, visual content, proof obligation, or tool-result correctness.

Thus generation is not arbitrary sampling. It is approximate search inside a policy-constrained continuation space.

11.8 Training as quotient formation

Let L_θ be a training loss.

Training updates θ so as to reduce L_θ over a corpus of presentations and transformations.

If L_θ penalizes failure to preserve task-relevant invariants, then optimization forms approximate equivalence classes in representation space.

Thus training induces learned policies $\pi\theta$ and approximate quotients

$\Omega/\pi\theta$.

Generalization corresponds to stability of these quotients under unseen perturbations.

11.9 Robustness

Let P be a perturbation class.

A learned policy $\pi\theta$ is robust under P if

$X \equiv\pi\theta\ Y \Rightarrow pX \equiv\pi\theta\ pY$

for all $p \in P$,

or if

$V\pi\theta(pX,pY) \leq V\pi\theta(X,Y)+\eta$

for small η.

Prompt robustness, paraphrase robustness, formatting robustness, and translation robustness are instances of policy-stability.

11.10 Capability as stable quotient

A capability C is a stable quotient if there exists policy πC such that:

inputs expressing the same task fall into common πC-orbits,

valid outputs preserve $Inv\pi C$,

iterative transformation decreases $V\pi C$,

and perturbations remain within $B\pi C$.

Thus a capability is not a single behavior. It is a stable admissibility basin.

11.11 Hallucination as certificate failure

Let Y be a generated output for input X under policy π.

A hallucination occurs when Y is accepted by the generative transformation but lacks a valid certificate:

$Y \in T\theta(X)$

but

$Cert\pi(X,Y)$ is empty

or

$V\pi(X,Y) > \theta\pi$.

Thus hallucination is not merely falsehood. It is admissibility failure relative to the governing policy.

11.12 Alignment as policy refinement

Let $\pi base$ be a base generative policy.

Let $\pi align$ be an aligned policy.

Alignment is policy refinement when

$\pi align \leq \pi base$.

Thus

$X \equiv \pi align\ Y \Rightarrow X \equiv \pi base\ Y$,

but not conversely.

Alignment shrinks the admissible orbit by adding constraints.

It is therefore a meet-like operation:

$\pi align = \pi base \wedge \pi safety \wedge \pi instruction \wedge \pi truth$

when those meets exist.

11.13 Tool use as external certification

Let τ be a tool.

A tool-mediated certificate has the form

$c = (Y, \tau, r, h)$

where Y is a generated candidate, τ is an external verifier or executor, $r=\tau(Y)$ is the result, and h certifies that r satisfies $Inv\pi$.

Thus tools convert internal approximate admissibility into externally certified admissibility.

Tool use is therefore certificate extension.

11.14 Approximate Lyapunov condition

Let $V\theta,\pi$ be policy-tension during inference, self-correction, or iterative refinement.

A process is approximately π-stable if

$E[V\theta,\pi(X_{n+1})] \leq E[V\theta,\pi(X_n)] + \eta$

with η small.

It is asymptotically π-correcting if

$E[V\theta,\pi(X_n)] \to 0$

or below an admissibility threshold.

Thus iterative refinement may be modeled as descent in learned policy-tension.

11.15 Learned emergence

A learned system exhibits emergent capability under π when:

π was not stably represented below parameter/data/compute threshold λc,

π becomes represented above λc,

Ω/π becomes nontrivial,

$V\pi$ has a stable basin,

and perturbations remain in the basin.

Thus machine emergence is stable learned quotient formation.

12. Policy Stacks and Multilevel Admissibility

12.1 Policy hierarchy

Let

$\pi_0, \pi_1, \ldots, \pi_n$

be policies on levels

$\Omega_0, \Omega_1, \ldots, \Omega_n$.

Let

$qi : \Omega_i \to \Omega_i/\pi_i$

be quotient maps.

A policy stack is a sequence of admissibility reductions and lifts

$\Omega_0 \to \Omega_0/\pi_0 \simeq \Omega_1 \to \Omega_1/\pi_1 \simeq \Omega_2 \to \ldots \to \Omega_n/\pi_n$.

Each level quotients lower-level variation and supplies presentations for the next level.

12.2 Nested quotient

The iterated quotient is

$\Omega_o/(\pi_o;\pi_1;...;\pi_n)$

defined when each quotient supplies the presentation space for the next policy.

This is not generally equal to quotienting by a single flattened policy.

Flattening loses intermediate certificates, stabilizers, and obstructions.

Thus policy stacks preserve emergence history.

12.3 Descent and lift

Given $q\pi : \Omega \to \Omega/\pi$, a structure S on Ω descends if there exists $S\pi$ on Ω/π such that

$q\pi_{(}{}^{*}{}_{)}S = S\pi$.

A structure R on Ω/π lifts if there exists $\tilde{R}$ on Ω such that

$q\pi_{(}{}^{*}{}_{)}\tilde{R} = R$.

Obstructions to descent or lift measure incompatibility between levels.

12.4 Interference of policies

Policies π and ρ interfere if their meet exists but is unstable, or if their join collapses, or if descent under one destroys invariant structure required by the other.

Formally, interference occurs if:

$\pi \wedge \rho$ exists but $B\pi\wedge\rho = \emptyset$,

or

$\pi \vee \rho$ collapses Ω,

or

$Inv\pi$ fails to descend along $q\rho$.

Thus multilevel systems require compatibility of policies.

12.5 Residuals

Given a projection policy π and a field F on Ω, define the π-residual

$R\pi(F) = F - P\pi F$

when a projection $P\pi$ exists.

Residuals measure structure not admitted by π.

At level transitions, residuals may become noise, obstruction, or seed data for a new policy.

Thus emergence may be driven by residual structure under prior admissibility.

12.6 Stable stack

A policy stack is stable if each level has a Lyapunov functional $V\pi_i$ and basin $B\pi_i$ such that descent from level i to i+1 preserves stability:

$x \in B\pi_i \Rightarrow qi(x) \in B\pi_{i+1}$.

Equivalently, there are monotone maps

$V\pi_{i+1}(qi(x)) \leq F_i(V\pi_i(x))$

for control functions F_i.

12.7 Collapse of stack

A policy stack collapses if some quotient is terminal, or if some join of level policies identifies all presentations, or if necessary invariants fail to descend.

Collapse at level k eliminates distinguishability needed by higher levels.

Thus noncollapse must be checked locally at each layer.

12.8 Stack emergence

An object E_n at level n is stack-emergent if it is obtained by iterated nontrivial stable quotients and cannot be reconstructed from any single lower-level representative without the intermediate policy data.

Thus stack-emergence strengthens stable quotient emergence by requiring irreducibility to flattened presentation.

12.9 Composition theorem

Theorem 12.9.1. Suppose $\Omega_0 \to \Omega_1 \to \ldots \to \Omega_n$ is a stable policy stack. If each quotient qi is nontrivial, each level admits a Lyapunov functional $V\pi_i$, and stability descends along qi, then the terminal object E_n is stable relative to the composite policy stack.

Proof. Induct on n. For n=1 this is stable quotient formation. Assume true for n−1. Stability at level n−1 descends to level n by hypothesis. Since q_n is nontrivial and $V\pi_n$ stabilizes E_n, the

terminal quotient is stable. Nontriviality at each stage prevents collapse. Therefore E_n is stable under the composite stack.

12.10 Non-flattening corollary

If some intermediate quotient has nontrivial stabilizer, obstruction, or residual data used by a higher policy, then the stack is not equivalent to a single quotient Ω_o/π.

Proof. A single flattened quotient records only final equivalence classes. It discards intermediate stabilizer, obstruction, and residual data unless explicitly enriched to retain them. If higher policy depends on such data, flattening is not faithful.

12.11 Interpretation

Policy stacks formalize multilevel object formation.

They are suitable for systems in which lower-level variation is repeatedly quotiented, stabilized, and re-presented at higher levels.

The mathematical content is not hierarchy alone, but certified descent of identity through nontrivial stable quotients.

13. Relation to Standard Structures

13.1 Sets

Let Ω be a set and π an equivalence relation.

Define

$\mathrm{Cert}\pi(X,Y) = \{*\}$ if $X \equiv\pi Y$,

$\mathrm{Cert}\pi(X,Y) = \emptyset$ otherwise.

Then the admissibility quotient Ω/π is the ordinary quotient set.

Thus set-level admissibility algebra reduces to equivalence relations and quotient sets.

The novelty begins when π is varied, refined, composed, certified, directed, or stabilized.

13.2 Preorders

Let Ω be a preorder.

Let π be the reachability policy:

$X \equiv\pi Y$ iff $X \leq Y$.

Then $\mathrm{Cert}\pi(X,Y)$ is inhabited when Y is reachable from X.

This policy is generally not symmetric.

Thus admissibility identity need not be equality-like unless symmetry is imposed.

Directed admissibility yields process categories rather than groupoids.

13.3 Categories

Let C be a category.

Let πmor be the morphism policy:

X ≡πmor Y iff HomC(X,Y) is inhabited.

Let πiso be the isomorphism policy:

X ≡πiso Y iff IsoC(X,Y) is inhabited.

Let πequiv be an equivalence policy in a bicategory or ∞-category.

Then

πiso ≤ πmor

when every isomorphism is a morphism.

The category C by itself does not determine which of these policies carries identity force.

Admissibility Algebra separates morphism existence from identity license.

13.4 Group actions

Let G act on Ω.

Define

X ≡G Y ⇔ ∃g ∈ G such that gX=Y.

Then

OrbG(X) = G·X,

StabG(X) = {g ∈ G : gX=X},

Ω/G is the orbit quotient.

Thus group-action orbit theory is an admissibility system with policy πG.

The admissibility generalization allows:

partial actions,

semigroup actions,

higher group actions,

directed actions,

context-indexed actions,

certified actions,

and Lyapunov-stable orbit formation.

13.5 Groupoids

Let G be a groupoid acting on Ω.

Then Certπ(X,Y) is the groupoid of arrows taking X to Y.

The quotient is an orbit groupoid or stack quotient.

Thus groupoid quotients appear as proof-relevant admissibility quotients.

Admissibility Algebra retains automorphism data through Stabπ(X).

13.6 Homotopy theory

Let Ω be a space or type.

Let πhom be the path policy:

X ≡πhom Y iff PathΩ(X,Y) is inhabited.

Let πheq be homotopy equivalence of spaces.

Then Certπ is path space or equivalence space.

The quotient Ω/π is a homotopy quotient when certificates are homotopical.

Admissibility Algebra differs from ordinary homotopy theory by making the choice of path policy variable rather than fixed.

13.7 Localization

Let C be a category and W a class of morphisms.

Let πW be the policy that declares W-morphisms identity-preserving.

Then the admissibility quotient is the localization

$C[W^{-1}]$.

If W is not closed or coherently invertible, one obtains partial localization or derived localization.

Thus localization is policy inversion.

The class W is the admissibility policy.

13.8 Congruences

Let A be an algebraic structure.

Let π be a congruence relation compatible with operations.

Then A/π is the ordinary algebraic quotient.

Scoped substitution corresponds exactly to compatibility of π with the operations of A.

Thus universal algebra is recovered when policies are congruences.

Admissibility Algebra generalizes congruence by allowing proof-relevant, directed, context-indexed, and stability-enriched policies.

13.9 Sheaves and stacks

Let X be a site.

Let F be a presheaf or stack.

A descent datum is an admissibility policy for gluing local presentations.

Local objects Fi are admissibly identical on overlaps when there exist certificates

cij : Fi|Uij → Fj|Uij

satisfying cocycle coherence

cjk ∘ cij = cik

on triple overlaps.

Failure of coherence is obstruction.

Thus descent is context-indexed admissibility.

Stacks are admissibility systems retaining automorphism and higher certificate data.

13.10 Gauge theory

Let Ω be a space of field presentations.

Let G be a gauge group.

Let πgauge identify field configurations in the same gauge orbit.

Then

Ω/πgauge = Ω/G

or, more accurately,

Ω//G

when stabilizers are retained.

Gauge-invariant observables are precisely πgauge-respecting contexts.

Gauge fixing is a section of the quotient where available.

Gauge anomalies are obstructions to coherent descent.

13.11 Renormalization

Let Ωλ be a space of descriptions at scale λ.

Let πλμ be an admissibility policy translating descriptions from scale λ to scale μ while preserving selected observables.

Then renormalization is a context-indexed admissibility system over a scale category.

A fixed point is a stable object under scale-translation policy.

Relevant, irrelevant, and marginal directions are classified by policy-relative Lyapunov or spectral behavior near scale-stable orbits.

13.12 Modal logic

A policy π can be read as a modality.

Define

□π P(X)

to mean P is invariant under π-admissible transformations of X.

Define

◇π P(X)

to mean P holds for some presentation in Orbπ(X).

Then refinement π ≤ ρ induces modal implication

□ρ P ⇒ □π P

for invariant predicates.

Strict policies make necessity stronger.

Loose policies make possibility larger.

Thus Admissibility Algebra has a modal internal language.

13.13 Type theory

Let $Id_\pi(X,Y)$ be a policy-indexed identity type.

Rules:

formation:

$X,Y : \Omega \vdash Id_\pi(X,Y)$ type

introduction:

$c : Cert_\pi(X,Y) \vdash c : Id_\pi(X,Y)$

elimination:

$p : Id_\pi(X,Y)$, C respects $\pi \vdash transport_{\pi,C}(p) : C(X) \to C(Y)$

computation:

$transport_{\pi,C}(refl_\pi) = id$

provided $refl_\pi$ exists.

Thus identity elimination is restricted by admissibility.

13.14 Summary

Standard structures appear as special admissibility regimes:

equivalence relation → set quotient

group action → orbit quotient

groupoid action → stack quotient

category with W → localization

congruence → algebraic quotient

homotopy → path quotient

gauge symmetry → gauge quotient

descent datum → sheaf/stack gluing

renormalization → scale-indexed policy transport

modal type theory → internal language of policy-indexed identity

The generalization is the explicit algebra of policy selection, refinement, composition, certification, and stability.

14. Algebra of Policies

14.1 Policy operations

Let $\mathfrak{P}$ be a locally ordered bicategory of policies.

For parallel policies $\pi,\rho : a \to b$, define:

$\pi \le \rho$ if π is stricter than ρ.

$\pi \wedge \rho$ if simultaneous admissibility exists.

$\pi \vee \rho$ if coherent generated admissibility exists.

$\pi \circ \rho$ when target and source contexts match.

$\pi\bot$ as the inconsistent policy, if present.

$\pi\top$ as the indiscrete collapsing policy, if present.

Not all operations are total.

The correct structure is therefore generally partial.

14.2 Policy meet

For parallel policies π,ρ, their meet satisfies

$\mathrm{Adm}\pi\wedge\rho = \mathrm{Adm}\pi \cap \mathrm{Adm}\rho$

when both are represented by admissible transformations.

Invariant structure is combined:

$\mathrm{Inv}\pi\wedge\rho = \mathrm{Inv}\pi \times \mathrm{Inv}\rho$.

Certificate structure is fiber product:

$\mathrm{Cert}\pi\wedge\rho(X,Y) = \mathrm{Cert}\pi(X,Y) \times \mathrm{Cert}\rho(X,Y)$

over shared coherence constraints.

Thus meet is intersection of identity licenses.

14.3 Policy join

The join $\pi \vee \rho$ is generated by admissible transformations of π and ρ.

Let $\langle \mathrm{Adm}\pi \cup \mathrm{Adm}\rho \rangle$ denote closure under identities, composition, and required coherence.

Then

$$\mathrm{Adm}\pi\vee\rho = \langle \mathrm{Adm}\pi \cup \mathrm{Adm}\rho \rangle$$

provided protected invariants remain nontrivial.

If closure forces all presentations equivalent, the join collapses.

If invariants conflict, the join fails.

Thus join is guarded closure.

14.4 Policy product

For independent presentation spaces Ω and Λ with policies π on Ω and ρ on Λ, define product policy $\pi \times \rho$ on $\Omega \times \Lambda$ by

$$(X,A) \equiv_{\pi\times\rho} (Y,B)$$

iff

$$X \equiv_\pi Y \text{ and } A \equiv_\rho B.$$

Then

$$\mathrm{Orb}_{\pi\times\rho}(X,A) = \mathrm{Orb}_\pi(X) \times \mathrm{Orb}_\rho(A).$$

If Lyapunov functions exist,

$$V_{\pi\times\rho}(X,A) = V_\pi(X) + V_\rho(A)$$

or any monotone aggregate.

14.5 Policy pullback

Let $F : \Omega \to \Lambda$.

Given a policy ρ on Λ, define the pullback policy $F^*\rho$ on Ω by

$$X \equiv_{F^*\rho} Y \iff F(X) \equiv_\rho F(Y).$$

Then F is automatically policy-preserving:

$X \equiv F^{*}\rho\ Y \Rightarrow F(X) \equiv \rho\ F(Y)$.

The quotient $\Omega/F^{*}\rho$ is the kernel quotient of F relative to ρ.

14.6 Policy pushforward

Given a policy π on Ω and a map $F : \Omega \to \Lambda$, define the pushforward policy $F!\pi$ on Λ as the smallest policy making F policy-preserving.

That is,

$F(X) \equiv F!\pi\ F(Y)$

whenever

$X \equiv \pi\ Y$,

closed under coherence.

Pushforward may collapse distinctions if F is nonfaithful.

14.7 Kernel policy

Given $F : \Omega \to \Lambda$ and policy ρ on Λ, the kernel policy is

$\ker\rho(F) = F^{*}\rho$.

Thus

$X \equiv \ker\rho(F)\ Y \Leftrightarrow F(X) \equiv \rho\ F(Y)$.

Ordinary kernel equivalence is the special case where ρ is equality.

14.8 Image policy

Given π on Ω and $F : \Omega \to \Lambda$, the image policy $\mathrm{im}\pi(F)$ is the restriction of $F!\pi$ to the image of F.

It records which outputs are identified because their preimages were π-identical.

If F is quotienting, $\mathrm{im}\pi(F)$ may be coarser than equality.

14.9 Policy exactness

A sequence

$\Omega \xrightarrow{F} \Lambda \xrightarrow{G} M$

is policy-exact at Λ if

$\mathrm{im}\pi(F) = \ker\sigma(G)$

for specified source policy π and target policy σ.

This generalizes exactness from algebra to admissibility.

Identity collapsed by G is exactly identity generated by F.

14.10 Policy adjunction

Let $F : \mathfrak{E}\pi \to \mathfrak{E}\rho$ and $G : \mathfrak{E}\rho \to \mathfrak{E}\pi$.

A policy adjunction $F \dashv G$ consists of certificate equivalences

$\mathrm{Cert}\rho(FX,Y) \simeq \mathrm{Cert}\pi(X,GY)$

natural in X,Y.

Thus adjunctions occur within or across policy fibers.

If F,G also preserve Lyapunov tension up to monotone bounds, the adjunction is dynamically admissible.

14.11 Policy monad

A policy monad T on a presentation category Ω consists of:

$T : \Omega \to \Omega$,

$\eta : \mathrm{id} \to T$,

$\mu : T^2 \to T$,

with a policy πT such that T(X) is the πT-closure of X.

The orbit $\mathrm{Orb}_{\pi T}(X)$ is represented by T(X).

Examples:

closure operators,

normalization,

completion,

localization,

gauge closure,

homotopy completion,

semantic closure.

The monad laws express coherent admissibility closure.

14.12 Policy comonad

A policy comonad G extracts admissible observable content.

G(X) is the invariant presentation visible under policy π.

Counit

$\varepsilon : G(X) \to X$

embeds observable/invariant content into the full presentation.

Comultiplication

$\delta : G(X) \to G^2(X)$

expresses stability of observation under repeated invariant extraction.

14.13 Policy closure

Given π, define closure of X by

$Cl\pi(X) = Orb\pi(X)$

or by the object representing Orbπ(X), when it exists.

Then

$X \in Cl\pi(X)$,

$Cl\pi(Cl\pi(X)) = Cl\pi(X)$,

$X \equiv\pi Y \Rightarrow Cl\pi(X)=Cl\pi(Y)$.

Thus admissibility orbits behave as closure operators when π is reflexive and transitive.

14.14 Boundary

Define the π-boundary of an orbit as the set of presentations adjacent to but not inside the orbit.

Given adjacency relation R on Ω,

$\partial\pi\ Orb\pi(X) = \{Y \notin Orb\pi(X) : \exists Z \in Orb\pi(X), R(Y,Z)\}$.

Boundary points are minimal identity failures.

In dynamical systems, crossing ∂π Orbπ(X) is identity rupture.

In finite graphs, boundary size measures local fragility.

14.15 Curvature and holonomy

In context-indexed systems, let policies π_{ij} translate between overlapping contexts U_i, U_j.

Given a cycle

$U_i \to U_j \to U_k \to U_i$,

the composite policy may differ from identity:

$\pi_{ki} \circ \pi_{jk} \circ \pi_{ij} \neq id_i$.

The deviation is policy holonomy.

If the induced certificate around the cycle fails to be trivial, there is admissibility curvature.

This captures obstruction to global identity gluing.

14.16 Policy obstruction

Let local certificates c_{ij} exist on overlaps.

A global object exists iff higher coherence certificates exist on all cycles and higher intersections.

Obstruction classes measure failure of such coherence.

Thus admissibility obstruction generalizes cocycle obstruction, anomaly, and contextual inconsistency.

14.17 Collapse theorem

Theorem 14.17.1. Let π and ρ be policies on Ω. Suppose $Adm_\pi \cup Adm_\rho$ generates a connected transformation graph on Ω and the generated join preserves no distinguishing invariant. Then $\pi \vee \rho$ collapses Ω.

Proof. Since the generated admissible graph is connected, every $X, Y \in \Omega$ are joined by a finite path of generated admissible transformations. Closure under composition gives $X \equiv_{\pi \vee \rho} Y$. Since no invariant distinguishes any pair, no certificate obstruction remains. Hence all presentations are identified.

14.18 Noncollapse theorem

Theorem 14.18.1. Let π be a policy on Ω. If there exists a π-invariant function $f : \Omega \to Z$ with at least two values, then π is noncollapsed.

Proof. If π collapsed Ω, all X,Y would satisfy X ≡π Y. Since f is π-invariant, f(X)=f(Y) for all X,Y. This contradicts the assumption that f has at least two values.

Thus nonconstant invariants certify noncollapse.

14.19 Refinement theorem

Theorem 14.19.1. If π ≤ ρ and ρ is noncollapsed, π need not be noncollapsed. But if π ≤ ρ and π collapses, then ρ collapses.

Proof. If π collapses, then for all X,Y, X ≡π Y. Since π ≤ ρ, X ≡ρ Y. Hence ρ collapses.

Corollary. Collapse is upward-closed in the weakening direction.

14.20 Compatibility

Two policies π,ρ are compatible if:

π ∧ ρ exists,

π ∨ ρ exists,

π ∨ ρ does not collapse,

and Invπ, Invρ have coherent common transport.

Compatibility is required for simultaneous identity preservation.

Incompatible policies produce obstruction, instability, or collapse.

15. The Internal Logic of Admissibility

15.1 Judgments

A policy-indexed logic has judgments

Γ ⊢ π policy

Γ ⊢ X : Ω

Γ ⊢ c : Certπ(X,Y)

Γ ⊢ X ≡π Y

The last is definable by inhabitation:

Γ ⊢ X ≡π Y iff Γ ⊢ c : Certπ(X,Y) for some c.

15.2 Reflexivity

A policy π is reflexive if for all X,

$\mathrm{refl}_{\pi,X} : \mathrm{Cert}_{\pi}(X,X)$

exists.

Not all candidate policies are reflexive.

Nonreflexive policies describe transitions rather than identity regimes.

15.3 Symmetry

A policy π is symmetric if there is an operation

$\mathrm{sym}_{\pi} : \mathrm{Cert}_{\pi}(X,Y) \to \mathrm{Cert}_{\pi}(Y,X)$.

Directed policies lack symmetry.

In irreversible systems, identity transport may be forward-valid but not backward-valid.

15.4 Transitivity

A policy π is transitive if there is composition

$\mathrm{comp}_{\pi} : \mathrm{Cert}_{\pi}(X,Y) \times \mathrm{Cert}_{\pi}(Y,Z) \to \mathrm{Cert}_{\pi}(X,Z)$

satisfying associativity up to Coh_{π}.

If π is reflexive, symmetric, and transitive, it is equivalence-like.

If only reflexive and transitive, it is preorder-like.

If composition is partial, it is groupoid-like or category-like only locally.

15.5 Identity rules

Formation:

$$\frac{\Gamma \vdash_{\pi} X,Y : \Omega}{\Gamma \vdash \mathrm{Id}_{\pi}(X,Y)\ \mathrm{type}}$$

Introduction:

$$\frac{\Gamma \vdash c : \mathrm{Cert}_{\pi}(X,Y)}{\Gamma \vdash c : \mathrm{Id}_{\pi}(X,Y)}$$

Elimination:

Γ ⊢ p : Idπ(X,Y)
Γ ⊢ C respects π with target σ
————————————————
Γ ⊢ transportC(p) : Idσ(CX,CY)

Computation:

transportC(reflπ,X) = reflσ,CX

when C preserves reflexivity.

15.6 Policy conversion

If α : π ≤ ρ, then there is a coercion rule

Γ ⊢ p : Idπ(X,Y)
Γ ⊢ α : π ≤ ρ
————————————————
Γ ⊢ α(p) : Idρ(X,Y)

Thus identity proofs can be weakened along policy refinement.

There is generally no coercion in the opposite direction.

15.7 Meet introduction and elimination

Meet introduction:

p : Idπ(X,Y), q : Idρ(X,Y)
————————————————————
(p,q) : Idπ∧ρ(X,Y)

Meet elimination:

r : Idπ∧ρ(X,Y)
————————————————————
π_1(r) : Idπ(X,Y)

r : Idπ∧ρ(X,Y)
————————————————————
π_2(r) : Idρ(X,Y)

Thus meet behaves as conjunction of identity licenses.

15.8 Join introduction

Join introduction:

$p : Id_{\pi}(X,Y)$

———————————————

$\iota_1(p) : Id_{\pi \vee \rho}(X,Y)$

$q : Id_{\rho}(X,Y)$

———————————————

$\iota_2(q) : Id_{\pi \vee \rho}(X,Y)$

Join elimination requires proving that the target context respects both π and ρ and is stable under generated closure.

Thus join elimination is guarded.

15.9 Collapse rule

If $\pi = \top$ is the indiscrete policy, then

$\Gamma \vdash Id_{\top}(X,Y)$

for all X,Y.

No nontrivial substitution follows unless the target context respects ⊤.

But a ⊤-respecting context is constant on Ω.

Therefore collapsed identity has no discriminating substitution power.

15.10 Invariant predicate

A predicate P on Ω is π-invariant if

$Id_{\pi}(X,Y) \to (P(X) \leftrightarrow P(Y))$.

Equivalently, P factors through Ω/π.

Thus invariant predicates are predicates on the quotient.

15.11 Modal operators

Define

$\Box_{\pi} P(X) \Leftrightarrow \forall Y,\ X \equiv_{\pi} Y \Rightarrow P(Y)$.

Define

$\Diamond_{\pi} P(X) \Leftrightarrow \exists Y,\ X \equiv_{\pi} Y$ and $P(Y)$.

Then:

$\pi \leq \rho \Rightarrow \Box_\rho P(X) \rightarrow \Box_\pi P(X)$,

$\pi \leq \rho \Rightarrow \Diamond_\pi P(X) \rightarrow \Diamond_\rho P(X)$.

Necessity is contravariant in policy weakening.

Possibility is covariant in policy weakening.

15.12 Policy-dependent truth

A statement P(X) is stable under π if

$P(X) \Rightarrow \Box_\pi P(X)$.

A statement is fragile under π if P(X) holds but not $\Box_\pi P(X)$.

Thus truth may be presentation-level while invariance is policy-level.

Mathematical objectivity requires specifying which statements are stable under which policies.

15.13 Observables

An observable $O : \Omega \rightarrow Z$ is π-admissible if

$X \equiv_\pi Y \Rightarrow O(X)=O(Y)$.

Equivalently, O factors:

$\Omega \xrightarrow{q_\pi} \Omega/\pi \xrightarrow{\bar{O}} Z$.

Thus observables are quotient functions.

Gauge-invariant observables, homotopy invariants, algebraic invariants, and conserved quantities are instances.

15.14 Definability

A property P is definable at policy π if P factors through Ω/π.

A construction C is definable at policy π if C respects π.

A theorem is policy-stable if all its constructions and substitutions are valid under the stated policies.

15.15 Principle

No identity judgment is mathematically complete unless it states:

the policy π,

the admissible transformations Adm_π,

the protected invariant Invπ,

the certificate type Certπ,

the coherence laws Cohπ,

and the contexts in which substitution is licensed.

16. Open Problems

16.1 Classification of policy bicategories

Classify bicategories $\mathfrak{P}$ suitable as bases for admissibility fibrations.

Determine minimal axioms under which $\mathfrak{P}$ supports:

policy refinement,

partial meet,

guarded join,

composition,

collapse detection,

certificate transport,

contextual gluing,

Lyapunov enrichment.

Problem. Characterize when a policy bicategory is equivalent to a quantaloid, double category, locally ordered bicategory, modal base, or indexed ∞-topos base.

16.2 Existence of admissibility quotients

Given a policy π on Ω, determine conditions for existence of Ω/π.

Cases:

set quotient,

homotopy quotient,

stack quotient,

categorical localization,

directed quotient,

partial quotient,

filtered quotient,

probabilistic quotient.

Problem. Give necessary and sufficient conditions on Certπ and Cohπ for Ω/π to exist in a specified ambient ∞-category.

16.3 Collapse detection

Given policies π,ρ, determine whether π ∨ ρ exists and whether it collapses Ω.

Problem. Develop computable criteria for collapse in finite admissibility systems.

Problem. Develop obstruction theory for collapse in infinite or higher-categorical systems.

Problem. Classify noncollapsed joins.

16.4 Policy exactness

Develop a theory of exact sequences

Ω —F→ Λ —G→ M

with

imπ(F) = kerσ(G).

Problem. Determine whether standard homological algebra can be recovered from policy exactness under additive hypotheses.

Problem. Define derived functors of policy quotienting.

16.5 Policy-indexed univalence

Given Idπ(X,Y) := Certπ(X,Y), determine when the map

(X ≃π Y) → Idπ(X,Y)

is an equivalence.

Problem. Classify policies satisfying π-univalence.

Problem. Determine interaction between π-univalence and policy refinement π ≤ ρ.

Problem. Determine whether a universe can support multiple incompatible univalence policies simultaneously.

16.6 Scoped substitution calculus

Develop a formal proof theory for policy-scoped substitution.

Problem. Give introduction, elimination, computation, and conversion rules for Idπ.

Problem. Determine normalization and consistency results.

Problem. Compare policy-indexed identity with modal, guarded, cohesive, parametric, and cubical type theories.

16.7 Policy sheaves and stacks

Let $\mathfrak{P}$ be a context category or site.

Problem. Define admissibility sheaves whose transition functions are policies and whose gluing data are certificates.

Problem. Classify admissibility stacks by obstruction classes arising from failed certificate coherence.

Problem. Relate policy holonomy to contextuality, anomaly, descent failure, and curvature.

16.8 Lyapunov-enriched fibers

Given $p : \mathfrak{E} \to \mathfrak{P}$ and $V\pi : \mathrm{Obj}(\mathfrak{E}\pi) \to \mathbb{R}_{\geq 0}$, determine coherence conditions for policy transport.

Problem. For $\alpha : \pi \to \rho$, classify when

$V\rho(W\pi\rho X) \leq V\pi(X)$.

Problem. Develop a theory of enriched admissibility fibrations over ordered, metric, Lawvere-metric, or energy-enriched bases.

16.9 Stable quotient theorem

Formalize the theorem:

Emergence = Lyapunov-stable admissibility quotient formation.

Problem. State and prove the theorem in ordinary dynamical systems.

Problem. Extend to stochastic dynamics.

Problem. Extend to hybrid systems.

Problem. Extend to categorical dynamics.

Problem. Extend to ∞-categorical and stack-valued dynamics.

16.10 Rupture/emergence duality

Given policies π and ρ, define cross-policy transition by

Vπ ↑ beyond θπ,

Vρ ↓ toward 0.

Problem. Classify bifurcations where rupture under π is emergence under ρ.

Problem. Relate to phase transitions, symmetry breaking, renormalization, biological novelty, and learned capability thresholds.

16.11 Finite admissibility objects

Classify finite systems

$\mathcal{G}$ = (Ω, ϕ, D, Q, o)

with:

finite presentation space Ω,

symbolic quotient ϕ,

incidence projection D,

property assignment Q,

ordering/topology o.

Problem. Classify rank-deficient incidence policies.

Problem. Classify quotient-compatible graph orderings.

Problem. Determine extremal self-loop structures under pushforward multigraphs.

Problem. Determine when finite admissibility objects admit nontrivial Lyapunov tension.

16.12 Genetic-code classification

For the standard genetic-code object

$\mathcal{G}$std = ($\mathcal{C}$, ϕ, D, Q, o),

complete the finite classification of:

translation quotients,

rank-10 additive projections,

residual interaction fields,

ordered-cube policies,

self-loop extrema,

anisotropic alphabet-order triples,

codon-neighborhood tension spectra.

Problem. Determine whether G-A-C-U optimality is unique up to natural symmetry under specified objectives.

Problem. Determine the invariant content of rank(D)=10 among alternative genetic codes.

16.13 Approximate admissibility

Develop filtered policies $\pi\varepsilon$ indexed by $\varepsilon \geq 0$.

Problem. Relate filtered admissibility quotients $\Omega/\pi\varepsilon$ to persistent homology.

Problem. Define bottleneck distance between policy filtrations.

Problem. Classify stability of approximate identity clouds.

16.14 Probabilistic admissibility

Let $P\pi(X,Y)$ be a probability of admissibility.

Problem. Define probabilistic certificate spaces.

Problem. Develop Bayesian policy refinement.

Problem. Relate $V\pi = -\log P\pi$ to Lyapunov tension.

Problem. Define stochastic admissibility quotient Ω/π in measure-theoretic and categorical terms.

16.15 Learned admissibility

Given a learned system $A\theta$, classify task-conditioned policies $\pi\theta,k$.

Problem. Define semantic quotient formation in embedding space.

Problem. Define capability as stable quotient.

Problem. Define hallucination as certificate failure.

Problem. Define alignment as policy refinement.

Problem. Determine whether scaling thresholds correspond to stable emergence of new policy quotients.

17. Conclusion

17.1 Main construction

Admissibility Algebra is most naturally formulated as an indexed ∞-categorical theory over a policy bicategory.

The base $\mathfrak{P}$ contains policies, refinements, compositions, guarded joins, meets, and collapse phenomena.

The total category $p : \mathfrak{E} \to \mathfrak{P}$ contains policy-relative objects and certificate-bearing transformations.

The fiber $\mathfrak{E}\pi$ contains the identity regime licensed by π.

The primitive judgment is

$X \equiv_\pi Y \Leftrightarrow \mathrm{Cert}_\pi(X,Y)$ is inhabited.

Thus identity is not primitive equality. It is certified admissibility under policy.

17.2 Objecthood

The primitive object is not X.

The primitive object is

$X_\pi = (X, \pi, \mathrm{Orb}_\pi(X), \mathrm{Stab}_\pi(X), \mathrm{Inv}_\pi(X), \mathrm{Cert}_\pi, \mathrm{Coh}_\pi)$.

A presentation may generate multiple nonidentical policy-objects.

Thus objecthood is indexed by admissibility jurisdiction.

17.3 Quotient

The primitive quotient is not $\Omega/\approx$ for a fixed equivalence relation.

It is

Ω/π,

whose mathematical type depends on Cert_π:

set quotient,

homotopy quotient,

stack quotient,

localization,

directed quotient,

filtered quotient,

probabilistic quotient.

Policy quotienting is therefore a family of fiberwise colimit constructions.

17.4 Substitution

The primitive substitution rule is not global Leibniz substitution.

It is scoped substitution:

$X \equiv_\pi Y$ and C respects π imply $C[X] \equiv_\sigma C[Y]$.

Equivalence without policy does not license substitution.

Only certified admissibility in a compatible context licenses substitution.

17.5 Dynamics

The primitive stable object is not a point.

It is a policy-orbit.

A π-Lyapunov function satisfies

$V_\pi(X)=0 \Leftrightarrow X \in \mathrm{Orb}_\pi(E)$,

with

$\Delta V_\pi \leq 0$

or

$\dot{D}V_\pi \leq 0$

under admissible dynamics.

Thus stability is indexed by identity policy.

17.6 Emergence

An emergent object is a nontrivial stable quotient.

Let M be a microstate space, π a policy, $q_\pi : M \to M/\pi$ the quotient, Φ dynamics, and V_π a Lyapunov-tension functional.

An object $E \in M/\pi$ emerges when:

q_π is nontrivial,

Cert_π is inhabited on the relevant orbit,

Inv_π is preserved,

Φ descends to the quotient,

$V_\pi^{-1}(0) = q_\pi^{-1}(E)$,

and V_π is nonincreasing on a basin.

Thus

Emergence = Lyapunov-stable admissibility quotient formation.

17.7 Position relative to Cat/HoTT

Category theory studies structured morphism systems after arrows have been admitted.

Homotopy type theory studies identity paths after identity spaces have been admitted.

Admissibility Algebra studies the prior algebra of admission.

It asks:

which transformations are identity-preserving,

which invariants are protected,

which witnesses certify identity,

which coherences are required,

which substitutions are licensed,

which quotients are noncollapsed,

which policy-orbits are stable.

Thus Admissibility Algebra is not below Cat/HoTT.

It is a policy layer above them.

17.8 Final statement

The central mathematical object is:

$\mathcal{A}$ = ($\mathfrak{P}$, p : $\mathfrak{E}$ → $\mathfrak{P}$, Cert, Inv, Coh, V).

Here $\mathfrak{P}$ is the policy bicategory, p is the admissibility fibration, Cert is the certificate assignment, Inv is the invariant assignment, Coh is higher coherence, and V is Lyapunov enrichment.

In one line:

Admissibility Algebra is indexed ∞-category theory over a policy bicategory, with certificate-valued identity and Lyapunov-stable quotient formation.

18. References

1. Youvan, Douglas C. Admissibility Algebra: Identity Policies, Invariant-Preserving Transformation, and a New Abstract Algebra Above Category Theory and Homotopy Type Theory. July 2026. DOI: 10.13140/RG.2.2.18444.07044.
2. Youvan, Douglas C. Reality Translates Contexts: From the Genetic Code Lookup Table to an Admissibility Theory of Physics, Mind, and Meaning. July 2026. DOI: 10.13140/RG.2.2.28090.96967.
3. Youvan, Douglas C. The Genetic Code as an Admissibility Algebra: Translation Quotients, Rank-10 Physicochemical Projection, and the G-A-C-U Topological Policy. July 2026. DOI: 10.13140/RG.2.2.35221.28643.
4. Youvan, Douglas C. Admissibility Algebra: A Policy-Relative Foundation for Identity, Invariance, and Certified Transformation. July 2026. DOI: 10.13140/RG.2.2.28510.40006.
5. Youvan, Douglas C. Stable Being: Lyapunov Functions, Admissibility Algebra, and Emergence as Certified Attractor Formation. July 2026. DOI: 10.13140/RG.2.2.23084.01927.
6. Youvan, Douglas C. Machine Emergence as Lyapunov-Stable Admissibility Algebra: Context Translation, Symbolic Quotients, and the Sudden Rise of Artificial Intelligence. July 2026. DOI: 10.13140/RG.2.2.10291.39209.
7. Youvan, Douglas C. From Stars to Selfhood: Admissibility Algebra, Genetic Code Topology, and the Human Shape as a Cosmic Translation Object. July 2026. DOI: 10.13140/RG.2.2.33150.34881.
8. Mac Lane, Saunders. Categories for the Working Mathematician. Second edition. Springer, 1998.

9. Borceux, Francis. Handbook of Categorical Algebra. Volumes 1–3. Cambridge University Press, 1994.

10. Johnstone, Peter T. Sketches of an Elephant: A Topos Theory Compendium. Volumes 1–2. Oxford University Press, 2002.

11. Lurie, Jacob. Higher Topos Theory. Princeton University Press, 2009.

12. Lurie, Jacob. Higher Algebra. 2017.

13. Riehl, Emily. Category Theory in Context. Dover, 2016.

14. Riehl, Emily, and Verity, Dominic. Elements of ∞-Category Theory. Cambridge University Press, 2022.

15. Leinster, Tom. Higher Operads, Higher Categories. Cambridge University Press, 2004.

16. Baez, John C., and Lauda, Aaron D. Higher-Dimensional Algebra V: 2-Groups. Theory and Applications of Categories, 2004.

17. Benabou, Jean. Introduction to Bicategories. Reports of the Midwest Category Seminar. Springer, 1967.

18. Street, Ross. Fibrations in Bicategories. Cahiers de Topologie et Géométrie Différentielle Catégoriques, 1980.

19. Grothendieck, Alexander. Revêtements étales et groupe fondamental. Séminaire de Géométrie Algébrique du Bois Marie 1960–1961.

20. Jacobs, Bart. Categorical Logic and Type Theory. Elsevier, 1999.

21. Hofmann, Martin, and Streicher, Thomas. The Groupoid Interpretation of Type Theory. Twenty-Five Years of Constructive Type Theory, Oxford University Press, 1998.

22. The Univalent Foundations Program. Homotopy Type Theory: Univalent Foundations of Mathematics. Institute for Advanced Study, 2013.

23. Awodey, Steve. Type Theory and Homotopy. Epistemology versus Ontology, Springer, 2012.

24. Voevodsky, Vladimir. Univalent Foundations of Mathematics. IAS lectures and manuscripts, 2010–2014.

25. Shulman, Michael. Univalence for Inverse Diagrams and Homotopy Canonicity. Mathematical Structures in Computer Science, 2015.

26. Shulman, Michael. Brouwer's Fixed-Point Theorem in Real-Cohesive Homotopy Type Theory. Mathematical Structures in Computer Science, 2018.

27. Lawvere, F. William. Metric Spaces, Generalized Logic, and Closed Categories. Rendiconti del Seminario Matematico e Fisico di Milano, 1973.

28. Lawvere, F. William. Adjointness in Foundations. Dialectica, 1969.

29. Kelly, G. M. Basic Concepts of Enriched Category Theory. Cambridge University Press, 1982.

30. Bénabou, Jean. Fibered Categories and the Foundations of Naive Category Theory. Journal of Symbolic Logic, 1985.

31. Giraud, Jean. Cohomologie non abélienne. Springer, 1971.

32. Jardine, J. F. Simplicial Presheaves. Journal of Pure and Applied Algebra, 1987.

33. Moerdijk, Ieke. Introduction to the Language of Stacks and Gerbes. 2002.

34. May, J. Peter. A Concise Course in Algebraic Topology. University of Chicago Press, 1999.

35. Hatcher, Allen. Algebraic Topology. Cambridge University Press, 2002.

36. Whitehead, George W. Elements of Homotopy Theory. Springer, 1978.

37. Bredon, Glen E. Topology and Geometry. Springer, 1993.

38. Mac Lane, Saunders, and Moerdijk, Ieke. Sheaves in Geometry and Logic. Springer, 1992.

39. Weibel, Charles A. An Introduction to Homological Algebra. Cambridge University Press, 1994.

40. Gelfand, Sergei I., and Manin, Yuri I. Methods of Homological Algebra. Springer, 1996.

41. Quillen, Daniel. Homotopical Algebra. Lecture Notes in Mathematics 43. Springer, 1967.

42. Dwyer, W. G., and Kan, D. M. Simplicial Localizations of Categories. Journal of Pure and Applied Algebra, 1980.

43. Rezk, Charles. A Model for the Homotopy Theory of Homotopy Theory. Transactions of the American Mathematical Society, 2001.

44. Cisinski, Denis-Charles. Higher Categories and Homotopical Algebra. Cambridge University Press, 2019.

45. Baez, John C., and Muniain, Javier P. Gauge Fields, Knots and Gravity. World Scientific, 1994.

46. Nakahara, Mikio. Geometry, Topology and Physics. Second edition. CRC Press, 2003.

47. Kobayashi, Shoshichi, and Nomizu, Katsumi. Foundations of Differential Geometry. Volumes 1–2. Wiley, 1963–1969.

48. Abraham, Ralph, Marsden, Jerrold E., and Ratiu, Tudor. Manifolds, Tensor Analysis, and Applications. Springer, 1988.

49. Arnold, Vladimir I. Mathematical Methods of Classical Mechanics. Springer, 1989.

50. Lyapunov, A. M. The General Problem of the Stability of Motion. 1892. English translation, Taylor & Francis, 1992.

51. LaSalle, Joseph P. The Stability of Dynamical Systems. SIAM, 1976.

52. Khalil, Hassan K. Nonlinear Systems. Third edition. Prentice Hall, 2002.

53. Strogatz, Steven H. Nonlinear Dynamics and Chaos. Second edition. Westview Press, 2015.

54. Hirsch, Morris W., Smale, Stephen, and Devaney, Robert L. Differential Equations, Dynamical Systems, and an Introduction to Chaos. Academic Press, 2013.

55. Guckenheimer, John, and Holmes, Philip. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, 1983.

56. Temam, Roger. Infinite-Dimensional Dynamical Systems in Mechanics and Physics. Springer, 1997.

57. Katok, Anatole, and Hasselblatt, Boris. Introduction to the Modern Theory of Dynamical Systems. Cambridge University Press, 1995.

58. Edelsbrunner, Herbert, and Harer, John. Computational Topology: An Introduction. American Mathematical Society, 2010.

59. Carlsson, Gunnar. Topology and Data. Bulletin of the American Mathematical Society, 2009.

60. Ghrist, Robert. Barcodes: The Persistent Topology of Data. Bulletin of the American Mathematical Society, 2008.

61. MacQueen, John. Some Methods for Classification and Analysis of Multivariate Observations. Proceedings of the Fifth Berkeley Symposium, 1967.

62. Bishop, Christopher M. Pattern Recognition and Machine Learning. Springer, 2006.

63. Goodfellow, Ian, Bengio, Yoshua, and Courville, Aaron. Deep Learning. MIT Press, 2016.

64. Vaswani, Ashish, et al. Attention Is All You Need. Advances in Neural Information Processing Systems, 2017.

65. Brown, Tom B., et al. Language Models are Few-Shot Learners. Advances in Neural Information Processing Systems, 2020.

66. Kaplan, Jared, et al. Scaling Laws for Neural Language Models. arXiv, 2020.

67. Hoffmann, Jordan, et al. Training Compute-Optimal Large Language Models. arXiv, 2022.

68. Bender, Emily M., Gebru, Timnit, McMillan-Major, Angelina, and Shmitchell, Shmargaret. On the Dangers of Stochastic Parrots. FAccT, 2021.

69. Mitchell, Melanie. Artificial Intelligence: A Guide for Thinking Humans. Farrar, Straus and Giroux, 2019.

70. Hopfield, John J. Neural Networks and Physical Systems with Emergent Collective Computational Abilities. Proceedings of the National Academy of Sciences, 1982.

71. Hinton, Geoffrey E., and Sejnowski, Terrence J. Learning and Relearning in Boltzmann Machines. Parallel Distributed Processing, MIT Press, 1986.

72. Crick, Francis H. C. On Protein Synthesis. Symposia of the Society for Experimental Biology, 1958.

73. Nirenberg, Marshall W., and Matthaei, J. Heinrich. The Dependence of Cell-Free Protein Synthesis in E. coli upon Naturally Occurring or Synthetic Polyribonucleotides. Proceedings of the National Academy of Sciences, 1961.

74. Crick, Francis H. C., Barnett, Leslie, Brenner, Sydney, and Watts-Tobin, Richard J. General Nature of the Genetic Code for Proteins. Nature, 1961.

75. Woese, Carl R. Order in the Genetic Code. Proceedings of the National Academy of Sciences, 1965.

76. Volkenstein, M. V. Configurational Statistics of Polymeric Chains. Interscience, 1963.

77. Sneath, P. H. A. Relations between Chemical Structure and Biological Activity in Peptides. Journal of Theoretical Biology, 1966.

78. Haig, David, and Hurst, Laurence D. A Quantitative Measure of Error Minimization in the Genetic Code. Journal of Molecular Evolution, 1991.

79. Freeland, Stephen J., and Hurst, Laurence D. The Genetic Code Is One in a Million. Journal of Molecular Evolution, 1998.

80. Knight, Rob D., Freeland, Stephen J., and Landweber, Laura F. Rewiring the Keyboard: Evolvability of the Genetic Code. Nature Reviews Genetics, 2001.

81. Mathews, Christopher K., van Holde, K. E., Appling, Dean R., and Anthony-Cahill, Spencer J. Biochemistry. Fourth edition. Pearson, 2012.

82. Alberts, Bruce, et al. Molecular Biology of the Cell. Sixth edition. Garland Science, 2014.

83. Cover, Thomas M., and Thomas, Joy A. Elements of Information Theory. Second edition. Wiley, 2006.

84. Shannon, Claude E. A Mathematical Theory of Communication. Bell System Technical Journal, 1948.

85. Rosen, Robert. Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life. Columbia University Press, 1991.

86. Kauffman, Stuart A. The Origins of Order. Oxford University Press, 1993.

87. Holland, John H. Emergence: From Chaos to Order. Addison-Wesley, 1998.

88. Barabási, Albert-László. Network Science. Cambridge University Press, 2016.

89. Newman, Mark. Networks. Second edition. Oxford University Press, 2018.

90. Pearl, Judea. Causality. Second edition. Cambridge University Press, 2009.

The author has published over 5,600 AI-assisted papers at: https://www.researchgate.net/profile/Douglas-Youvan/research . Google Scholar has indexed these preprints, and if you want a particular reference, the AI mode of a Google search (Gemini) has efficiently catalogued these preprints.